Superfluid BKT transition of 2D bubble-trapped condensates

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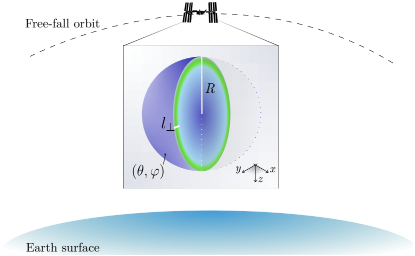

In collaboration with
A. Pelster, L. Salasnich.

This presentation is on
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Research on atomic Bose-Einstein condensates is driven by the *continuously-renewed experimental capability* of engineering new interactions and **trap configurations**.

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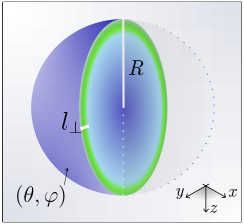
Shell-shaped BEC



curved and hollow 2D Bose gas

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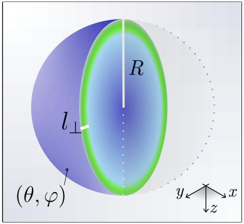
Shell-shaped BEC



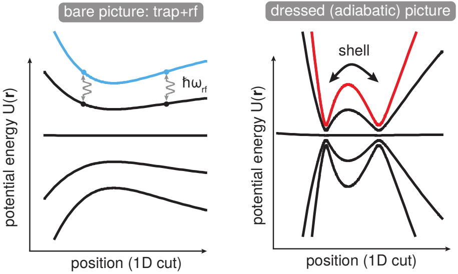
curved and hollow 2D Bose gas

In this talk, for BEC shells:

- Superfluid BKT transition
- Hydrodynamic excitations
- Thermodynamics

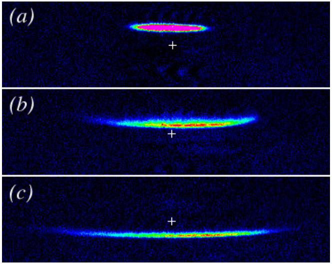
Experimentally realizable...in microgravity

Bubble-trap...



[Lundblad *et al.*, npj Microgravity **5**, 30 (2019)]

...on Earth

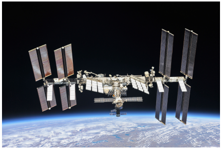


[Colombe *et al.*, EPL **67**, 593 (2004)]

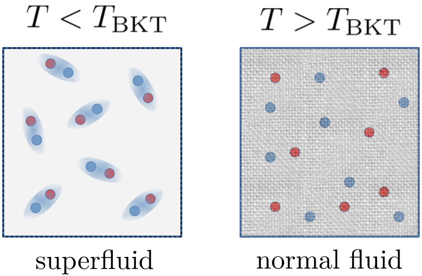
⇒ Experiments on NASA-JPL **Cold Atom Lab**

[Elliott *et al.*, npj Microgravity **4**, 16 (2018)]

[Aveline *et al.*, Nature **582**, 193 (2020)]



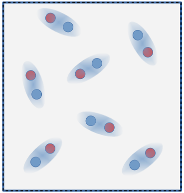
Berezinskii-Kosterlitz-Thouless transition – infinite flat case



BKT mechanism:
unbinding of vortex-antivortex dipoles at $T = T_{\mathrm{BKT}}$ suppresses the superfluidity

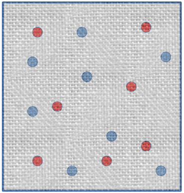
Berezinskii-Kosterlitz-Thouless transition – infinite flat case

$T < T_{\mathrm{BKT}}$



superfluid

$T > T_{\mathrm{BKT}}$



normal fluid

BKT mechanism:
unbinding of vortex-antivortex dipoles at $T = T_{\mathrm{BKT}}$ suppresses the superfluidity

Adimensional parameters

$K(\ell) = \frac{\hbar^2 n_s(\ell)}{m k_B T}$; $y(\ell)$

RG scale $\ell = \ln(r/\xi)$,

Distance between vortices: $r \in [\xi, \infty]$

[Nelson, Kosterlitz, PRL **39**, 1201 (1977)]

RG equations of a flat superfluid

$$\frac{dK^{-1}(\ell)}{d\ell} = -4\pi^3 y^2(\ell)$$

$$\frac{dy(\ell)}{d\ell} = [2 - \pi K(\ell)]\, y(\ell)$$

$\rightarrow$ From bare $n_s(\ell = 0) = n_s^{(0)}$
to renormalized $n_s = n_s(\ell = \infty)$

Is the vortex-antivortex unbinding the driving BKT mechanism also in shell-shaped condensates?

BKT transition – shell-shaped BECs

Let us assume the same mechanism and derive the consequences

RG equations of a spherical superfluid

$$\frac{dK^{-1}(\theta)}{d\ell(\theta)} = -4\pi^3 y^2(\theta)$$

$$\frac{dy(\theta)}{d\ell(\theta)} = [2 - \pi K(\theta)]\, y(\theta)$$

RG scale?

$$\ell(\theta) = \ln[2R\sin(\theta/2)/\xi]$$

Distance between vortices:

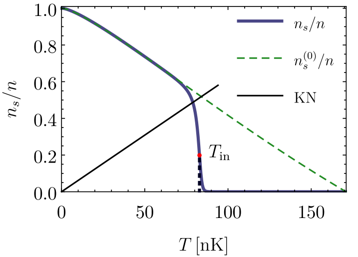
$2R\sin(\theta/2) \in [\xi, 2R]$...

...but in 3D space!!

[AT, Pelster, Salasnich, arXiv:2104.04585]

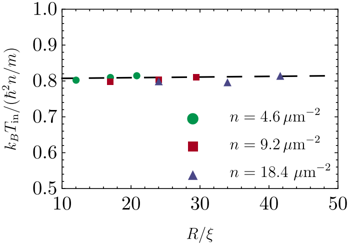
BKT transition – shell-shaped BECs

Finite system size ⇒ **smooth** vanishing of n_s

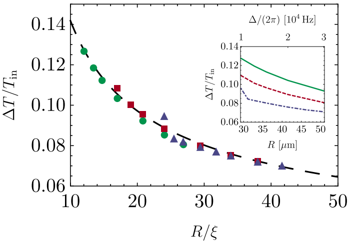


and finite-size BKT scaling:

$T_{\mathrm{in}} \sim n$



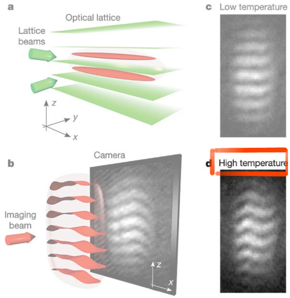
$\Delta T/T_{\mathrm{in}} \propto \ln^{-2}(R/\xi)$



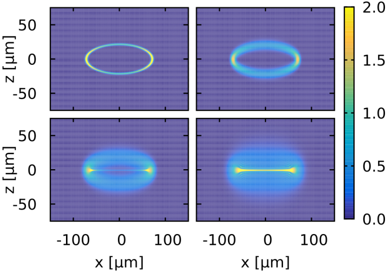
[AT, Pelster, Salasnich, arXiv:2104.04585]

Qualitative proof of BKT in shells

In flat superfluids:
vortex proliferation at T_{BKT}
$\Rightarrow$ "wavy" interference pattern



In superfluid shells,
free expansion at $T = 0$



...and a "wavy" pattern at T_{BKT}

[Hadzibabic et al. Nature **441**, 1118 (2006)]

[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

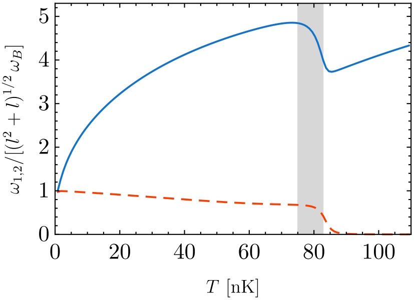
But how can we study quantitatively the BKT transition?

Hydrodynamic modes

Response of a finite-temperature superfluid to a small perturbation:

Flat case:
ordinary **first and second sound**
(basis: plane waves $e^{i(kx-\omega t)}$)

Shell BECs:
hydrodynamic modes ω_1, ω_2
(basis: spherical harmonics $\mathcal{Y}_l^{m_l} e^{i\omega t}$)



ω_1, ω_2 are the main quantitative probe of BKT physics

[AT, Pelster, Salasnich, arXiv:2104.04585]

Thermodynamics

Following [AT, Salasnich, PRL **123**, 160403 (2019)], we calculate the renormalized grand potential

$$\frac{\Omega}{4\pi R^2} = -\frac{m\mu^2}{8\pi\hbar^2}\left[\ln\left(\frac{4\hbar^2}{m(E_1^B+\epsilon_1+\mu)a^2\, e^{2\gamma+1}}\right)+\frac{1}{2}\right]$$

$$+\frac{mE_1^B}{8\pi\hbar^2}(E_1^B-\epsilon_1-\mu)+\frac{1}{4\pi R^2}\frac{1}{\beta}\sum_{l=1}^{\infty}\sum_{m_l=-l}^{l}\ln\left(1-e^{-\beta E_l^B}\right)$$

Thermodynamics

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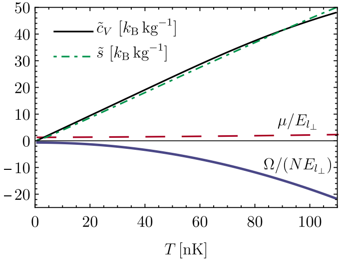
from which we calculate all the thermodynamic functions



While the hydrodynamic excitations are non-monotonic around T_{BKT}, the thermodynamic functions are unaffected by BKT

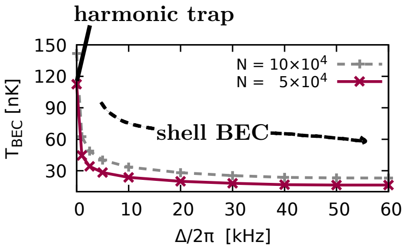
[AT, Pelster, Salasnich, arXiv:2104.04585]

Experimental relevance of finite-temperature properties

Are these predictions experimentally relevant? Yes!

For the realistic trap parameters of NASA-JPL CAL experiment:

T_{BEC} drops quickly
with $\Delta \propto$ shell area



[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

Difficult to reach fully-condensate regime...
⇒ Finite-temperature properties and BKT physics are highly relevant

In conclusion

We assume the vortex-antivortex unbinding as the driving BKT mechanism in shell-shaped condensates,

and derive the observable consequences:

– “wavy” imaging pattern

– hydrodynamic modes (vs continuous thermodynamics)

Finite-size BKT $\leftrightarrow$ curvature of quantum gases

Shell BECs $\leftrightarrow$ platform to study finite-size BKT

Thank you for your attention!

References

A. Tononi, F. Cinti, and L. Salasnich, *Quantum Bubbles in Microgravity*, Physical Review Letters **125**, 010402 (2020).

A. Tononi and L. Salasnich, *Bose-Einstein Condensation on the Surface of a Sphere*, Physical Review Letters **123**, 160403 (2019).

A. Tononi, A. Pelster, and L. Salasnich, arXiv:2104.04585