Quantum Statistics of a Shell-Shaped Bose-Einstein Condensate

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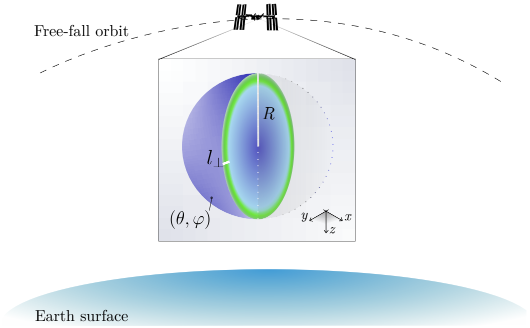
APS March Meeting 2021



In collaboration with
F. Cinti, A. Pelster, L. Salasnich.

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The physical properties of quantum gases depend on dimensionality, interactions, topology, infinite/finite size, **trap configuration**.

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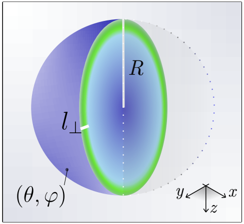
Shell-shaped BEC



topologically-nontrivial 2D curved Bose gas

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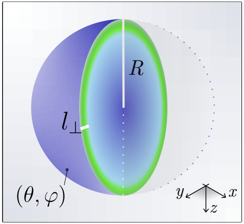
Shell-shaped BEC



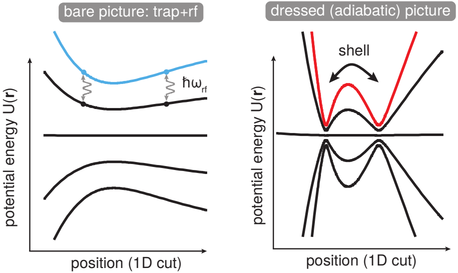
topologically-nontrivial 2D curved Bose gas

In this talk, for BEC shells:

- Superfluid BKT transition
- Hydrodynamic excitations
- Thermodynamics

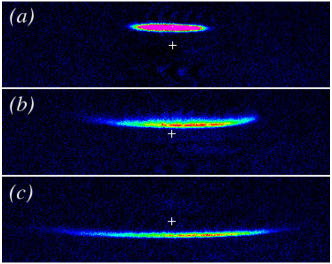
Experimentally realizable...in microgravity

Bubble-trap...



[Lundblad *et al.*, npj Microgravity **5**, 30 (2019)]

...on Earth

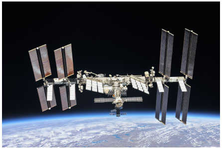


[Colombe *et al.*, EPL **67**, 593 (2004)]

⇒ Experiments on NASA-JPL **Cold Atom Lab**

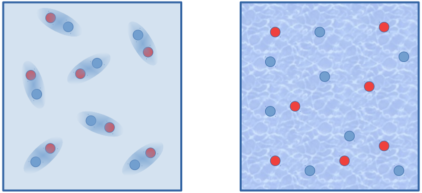
[Elliott *et al.*, npj Microgravity **4**, 16 (2018)]

[Aveline *et al.*, Nature **582**, 193 (2020)]



Berezinskii-Kosterlitz-Thouless transition – infinite flat case

$T < T_{\mathrm{BKT}}$ $\qquad$ $T > T_{\mathrm{BKT}}$



BKT mechanism:
unbinding of vortex-antivortex dipoles at $T = T_{\mathrm{BKT}}$ destroys the superfluidity

RG equations of a flat superfluid,
RG scale $\ell = \ln(r/\xi) \in [0, \infty]$

(with $K(\ell) = \frac{\hbar^2 n_s(\ell)}{m k_B T}$; $\; y(\ell)$)

$$\frac{dK^{-1}(\ell)}{d\ell} = -4\pi^3 y^2(\ell)$$

$$\frac{dy(\ell)}{d\ell} = [2 - \pi K(\ell)]\, y(\ell)$$

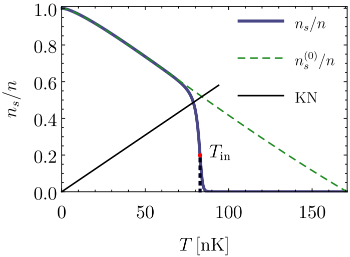
From bare $n_s(\ell = 0) = n_s^{(0)}$
to renormalized $n_s = n_s(\ell = \infty)$

[Nelson, Kosterlitz, PRL **39**, 1201 (1977)]

Is the vortex-antivortex unbinding the driving BKT mechanism also in shell-shaped condensates?

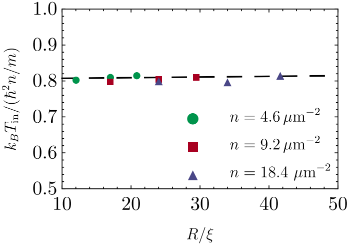
BKT transition – shell-shaped BECs

Extending the BKT theory to shell-shaped spherical BECs, we find **smooth** vanishing of n_s

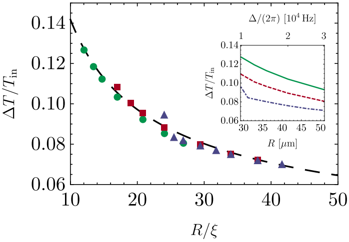


and universal laws in finite-size BKT:

$T_{\rm in} \propto n$



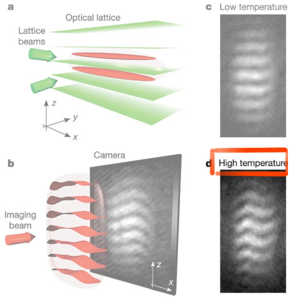
$\Delta T/T_{\rm in} \propto \ln^{-2}(R/\xi)$



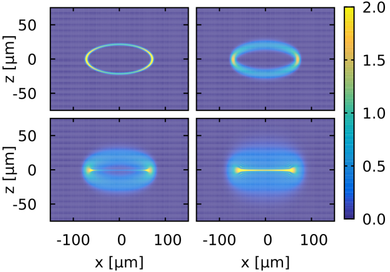
[AT, Pelster, Salasnich, *under review*]

Qualitative proof of BKT in shells

In flat superfluids:
vortex proliferation at $T_{\rm BKT}$
$\Rightarrow$ "wavy" interference pattern



In superfluid shells,
free expansion at $T = 0$



...and a "wavy" pattern at $T_{\rm BKT}$

[Hadzibabic et al. Nature **441**, 1118 (2006)] [AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

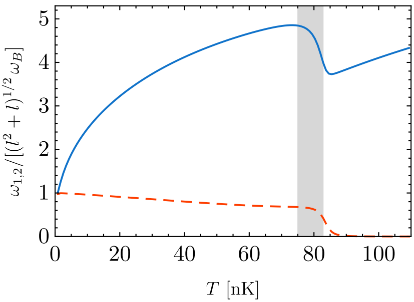
But how can we study quantitatively the BKT transition?

Hydrodynamic modes

Response of a finite-temperature superfluid to a small perturbation:

Flat case:
ordinary **first and second sound**
(basis: plane waves $e^{i(kx-\omega t)}$)

Shell BECs:
hydrodynamic modes ω_1, ω_2
(basis: spherical harmonics $\mathcal{Y}_l^{m_l} e^{i\omega t}$)

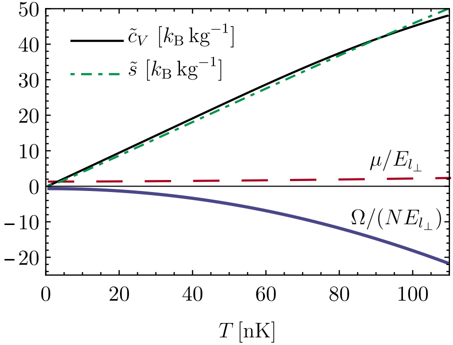


ω_1, ω_2 are the main quantitative probe of BKT physics

[AT, Pelster, Salasnich, *under review*]

Thermodynamics

Following our previous work [AT, Salasnich, PRL **123**, 160403 (2019)] , we calculate the renormalized grand potential Ω and we derive the various thermodynamic functions



While the hydrodynamic excitations are non-monotonic around T_{BKT}, the thermodynamic functions are unaffected by BKT

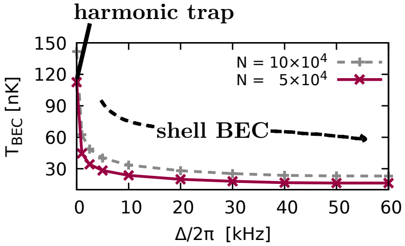
[AT, Pelster, Salasnich, *under review*]

Experimental relevance of finite-temperature properties

Are these predictions experimentally relevant? Yes!

For the realistic trap parameters of NASA-JPL CAL experiment:

T_{BEC} drops quickly with $\Delta \propto$ shell area



[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

Difficult to reach fully-condensate regime...

$\Rightarrow$ Finite-temperature properties and BKT physics are highly relevant

In conclusion

Is the vortex-antivortex unbinding the driving BKT mechanism also in shell-shaped condensates?

Yes, but we need experimental evidences:

- "wavy" imaging pattern
- hydrodynamic modes
- thermodynamics
- Universal laws in finite-size BKT, experimentally observable **in shell-shaped condensates**

Thank you for your attention!

References

A. Tononi, F. Cinti, and L. Salasnich, *Quantum Bubbles in Microgravity*, Physical Review Letters **125**, 010402 (2020).

A. Tononi and L. Salasnich, *Bose-Einstein Condensation on the Surface of a Sphere*, Physical Review Letters **123**, 160403 (2019).

A. Tononi, A. Pelster, and L. Salasnich, under peer review.