Topological BKT transition in bubble-trapped condensates

Andrea Tononi

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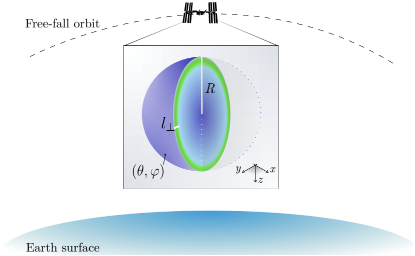
SuperFluctuations 2021



Based on

[AT, Pelster, Salasnich, arXiv:2104.04585]

This presentation on

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Research on atomic Bose-Einstein condensates is driven by the *continuously-renewed experimental capability* of engineering interatomic interactions and trap configurations.

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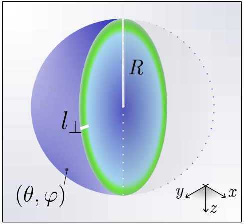
Bubble-trapped BEC



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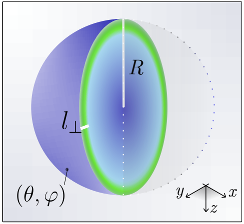
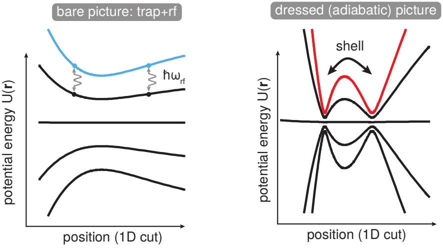


I will discuss:

- BKT transition: flat case vs spherical shell
- Hydrodynamic excitations, thermodynamics

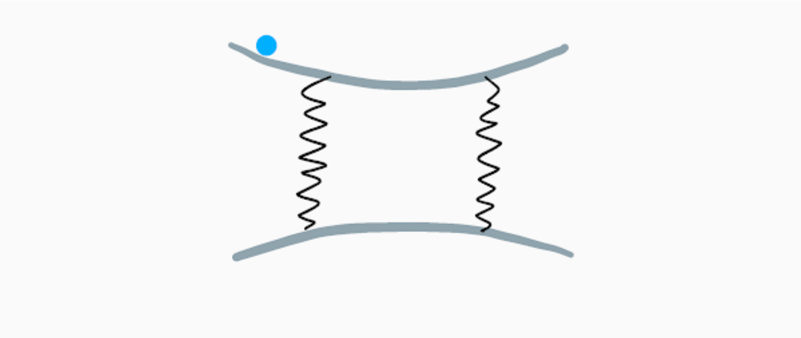
Experimentally realizable

Bubble-trap...



[Lundblad *et al.*, npj Micrograviy **5**, 30 (2019)]

Bubble-trap (rf-induced adiabatic potential)



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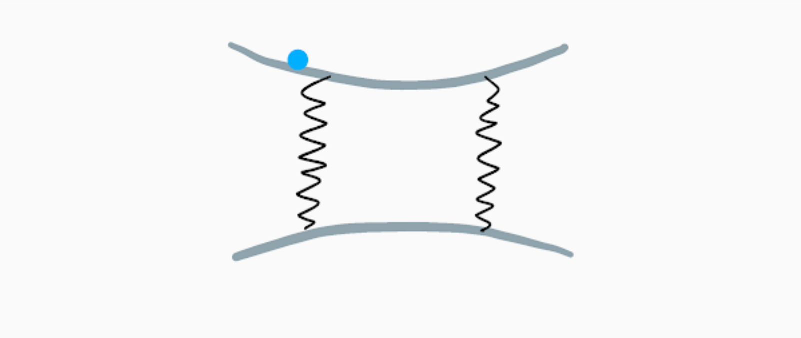

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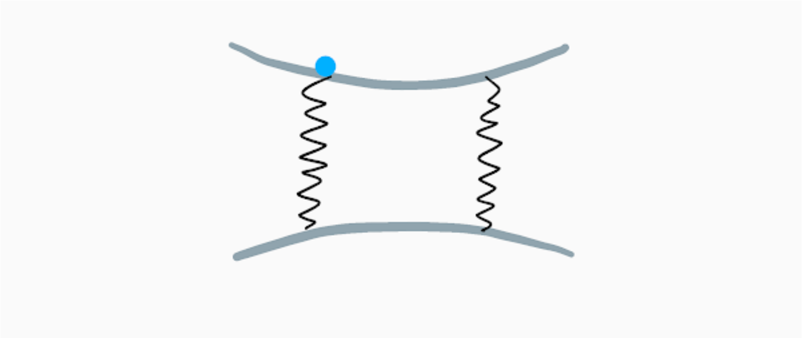

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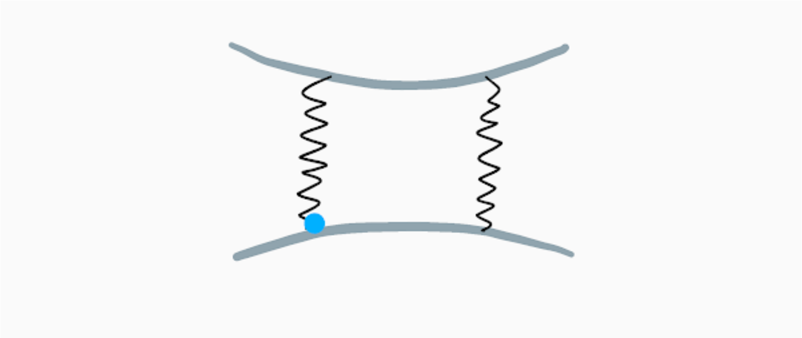

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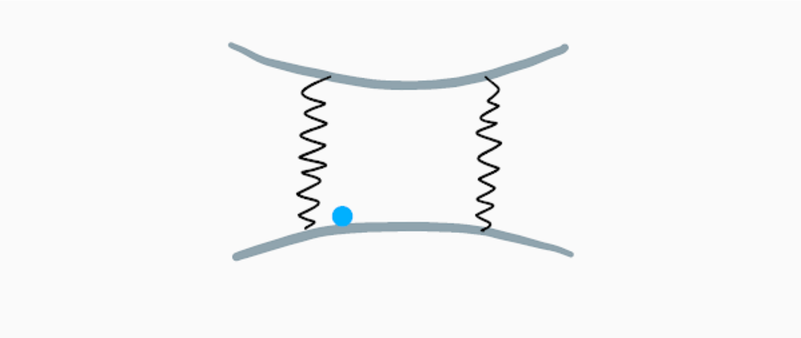

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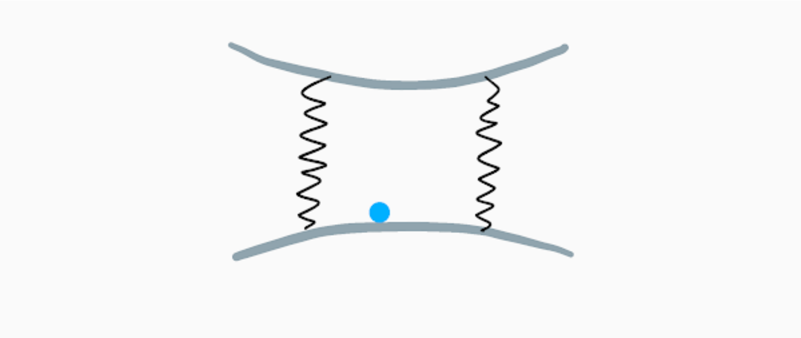

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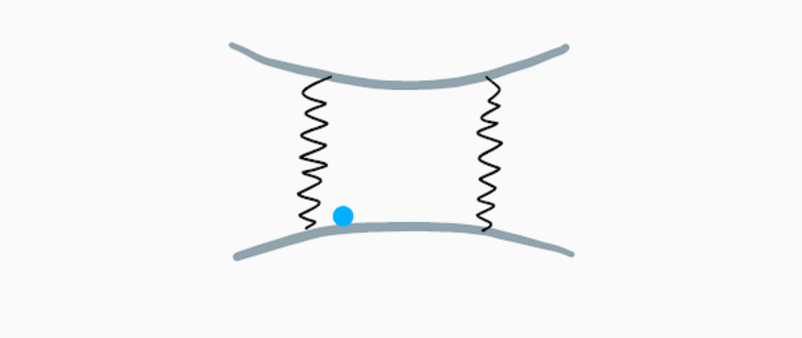

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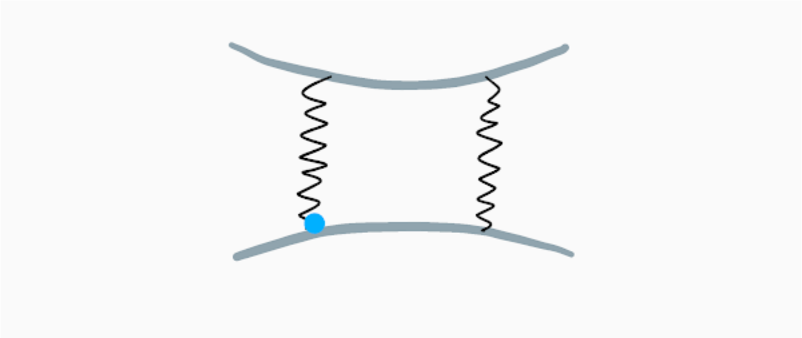

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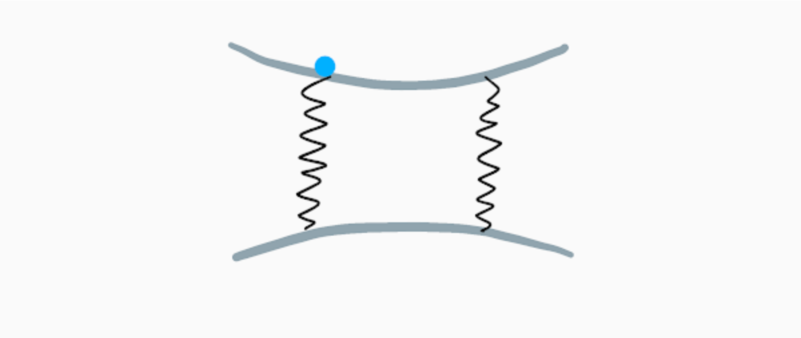

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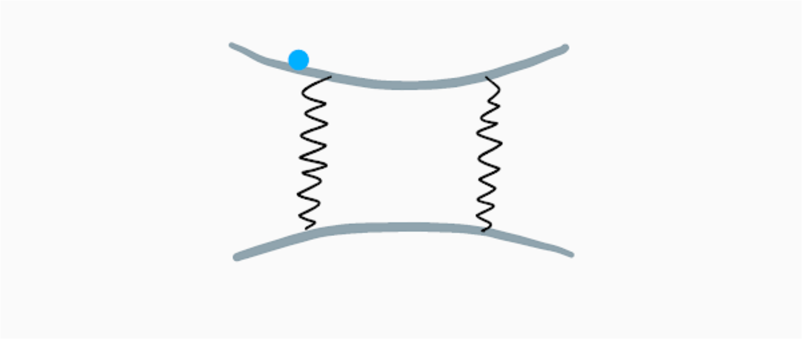

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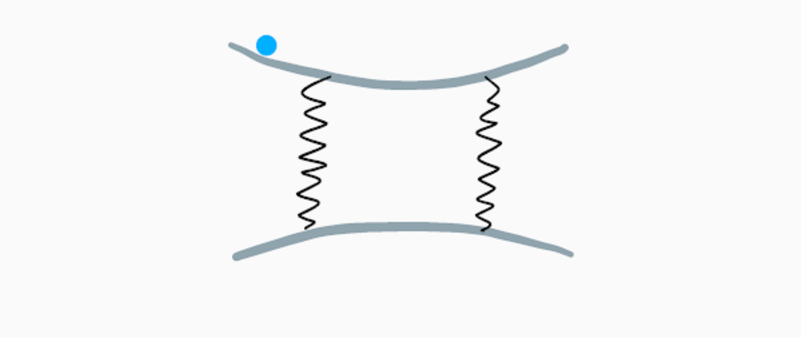
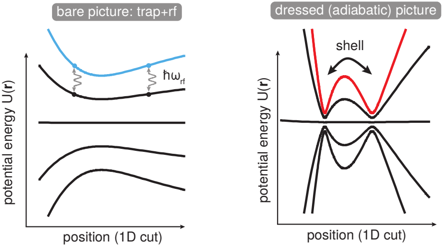


Bubble-trap (rf-induced adiabatic potential)

Experimentally realizable

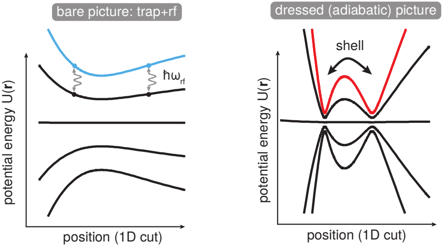
Bubble-trap...



[Lundblad *et al.*, npj Micrograviy **5**, 30 (2019)]

Experimentally realizable

Bubble-trap...



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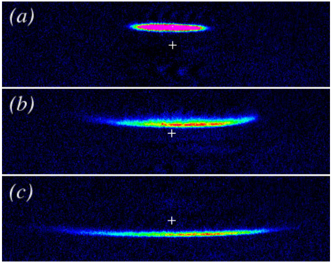
...on Earth



[Colombe *et al.*, EPL **67**, 593 (2004)]

Experimentally realizable

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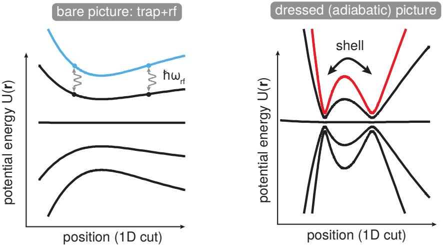

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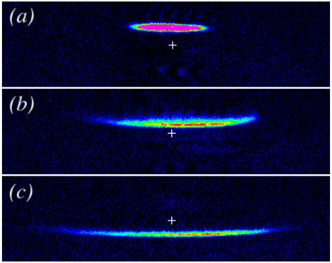
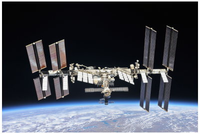


[Colombe *et al.*, EPL **67**, 593 (2004)]

⇒ Experiments on NASA-JPL **Cold Atom Lab**

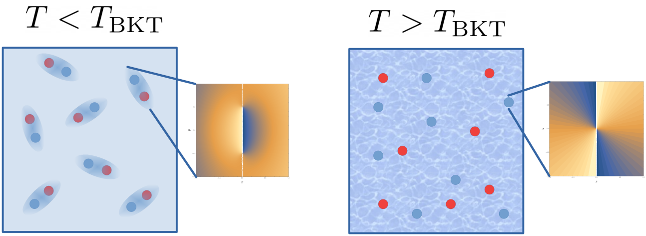
[Elliott *et al.*, npj Microgravity **4**, 16 (2018)]

[Aveline *et al.*, Nature **582**, 193 (2020)]



Berezinskii-Kosterlitz-Thouless transition – infinite flat case

Vortex-antivortex dipoles at $T < T_{\mathrm{BKT}}$, free vortices at $T > T_{\mathrm{BKT}}$



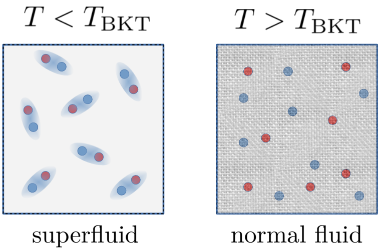
Simple model:

Free energy of a vortex in a 2D infinite superfluid:

$$F = U - TS = \frac{\pi \hbar^2 n_s^{(0)}(T)}{m} \ln\left(\frac{L}{\xi}\right) - T\, k_B \ln\left(\frac{L^2}{\xi^2}\right)$$

Vortices appear when $F < 0$, namely $T > T_{\mathrm{BKT}} = \frac{\pi \hbar^2 n_s^{(0)}(T)}{2 m k_B}$

Berezinskii-Kosterlitz-Thouless transition – infinite flat case



BKT mechanism:
unbinding of
vortex-antivortex dipoles
at $T = T_{\mathrm{BKT}}$ suppresses
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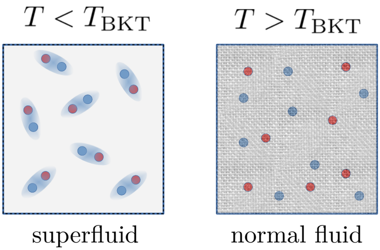

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Adimensional parameters

$K(\ell) = \frac{\hbar^2 n_s(\ell)}{m k_B T}$; $y(\ell)$

RG scale $\ell = \ln(r/\xi)$,

Distance between vortices:

$r \in [\xi, \infty]$

[Nelson, Kosterlitz, PRL **39**, 1201 (1977)]

RG equations of a flat superfluid

$$\frac{dK^{-1}(\ell)}{d\ell} = -4\pi^3 y^2(\ell)$$

$$\frac{dy(\ell)}{d\ell} = [2 - \pi K(\ell)]\, y(\ell)$$

$\to$ From bare $n_s(\ell = 0) = n_s^{(0)}$
to renormalized $n_s = n_s(\ell = \infty)$

BKT transition – bubble-trapped BECs

Is the superfluid transition driven by the vortex-antivortex unbinding also in shell-shaped condensates?

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Let us assume the same mechanism and derive the consequences

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RG equations of a spherical superfluid

$$\frac{dK^{-1}(\theta)}{d\ell(\theta)} = -4\pi^3 y^2(\theta)$$

$$\frac{dy(\theta)}{d\ell(\theta)} = [2 - \pi K(\theta)]\, y(\theta)$$

RG scale?

$$\ell(\theta) = \ln[2R\sin(\theta/2)/\xi]$$

Distance between vortices:

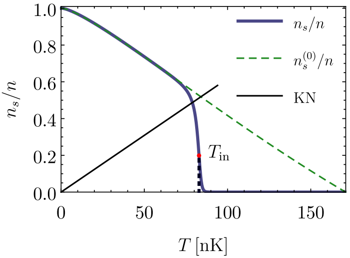
$2R\sin(\theta/2) \in [\xi, 2R]$...

...but in 3D space!!

[AT, Pelster, Salasnich, arXiv:2104.04585]

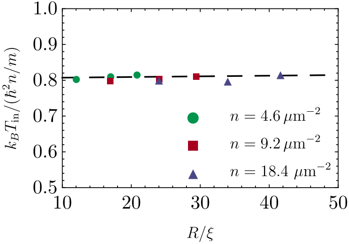
BKT transition – bubble-trapped BECs

Finite system size ⇒ **smooth** vanishing of n_s

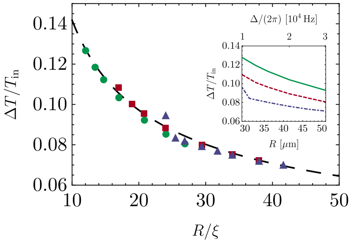


and finite-size BKT scaling:

$T_{\rm in} \sim n$



$\Delta T/T_{\rm in} \propto \ln^{-2}(R/\xi)$



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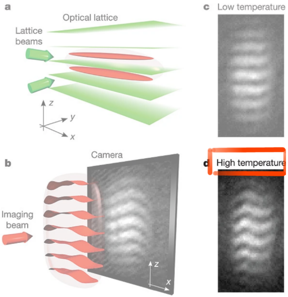
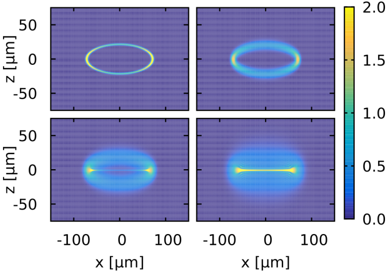
Qualitative proof of BKT in shells

In flat superfluids:
vortex proliferation at T_{BKT}
$\Rightarrow$ "wavy" interference pattern



[Hadzibabic et al. Nature **441**, 1118 (2006)]

In superfluid shells,
free expansion at $T = 0$



...and a "wavy" pattern at T_{BKT}

[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

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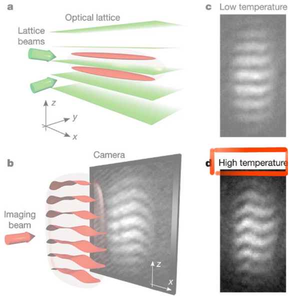

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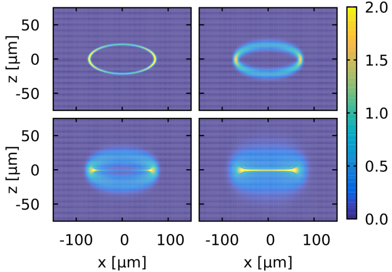

...and a "wavy" pattern at T_{BKT}

[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

But how can we study quantitatively the BKT transition?

Hydrodynamic modes – infinite flat case

Landau two-fluid model

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

$$\frac{\partial \rho \tilde{s}}{\partial t} + \rho \tilde{s} \, \nabla \cdot \mathbf{v}_n = 0$$

$$\frac{\partial \mathbf{j}}{\partial t} + \nabla P = 0$$

$$\frac{\partial \mathbf{v}_s}{\partial t} + \nabla \left(\frac{G_0}{M} \right) = 0$$

[Landau J. Phys. (USSR) **5**, 71 (1941)] see also [AT, et al. arXiv:2009.06491 accepted in PRA Letters] , [Furutani, AT, Salasnich, NJP **23**, 043043 (2021)]

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two fluids ⇒ two coupled sound equations

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 P$$
$$\frac{\partial^2 \tilde{s}}{\partial t^2} = \tilde{s}^2 \frac{n_s}{n_n} \nabla^2 T$$

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Expanding $\rho \sim \rho_0 + \rho' e^{i\omega(t - x/c)}$,

(equilibrium value ρ_0, fluctuation ρ')

and similarly for P, $\tilde{s}$, T...

[Landau J. Phys. (USSR) **5**, 71 (1941)] see also [AT, et al. arXiv:2009.06491 accepted in PRA Letters] , [Furutani, AT, Salasnich, NJP **23**, 043043 (2021)]

Hydrodynamic modes – infinite flat case

Landau two-fluid model predicts c_1, c_2, solutions of the biquadratic equation

$$c^4 - c^2\left[\left(\frac{\partial P}{\partial \rho}\right)_{\tilde{s}} + \frac{T\tilde{s}^2 n_s}{\tilde{c}_V n_n}\right] + \frac{n_s T\tilde{s}^2}{n_n \tilde{c}_V}\left(\frac{\partial P}{\partial \rho}\right)_T = 0$$

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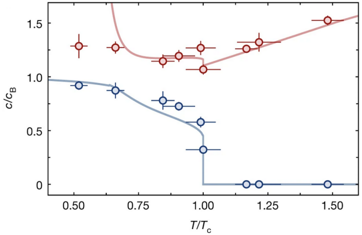
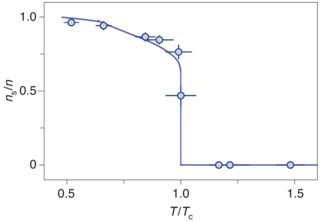


measure c_1, c_2,
find $T_c^{exp} = 42\,\mathrm{nK}$ experimentally,

theory for thermodynamics (scaled with $T_c^{th} = 37\,\mathrm{nK}$)



$\Rightarrow$ calculation of n_s/n

[Christodoulou, et al. Nature **594**, 191 (2021)]

Hydrodynamic modes – bubble-trapped BECs

Opposite path:

derive theoretically the thermodynamics and the superfluid density,

$\Rightarrow$ calculate the sound velocities... **Sound**??

plane waves is not the correct basis, but spherical harmonics $\mathcal{Y}_l^{m_l}$

$$(\rho \sim \rho_0 + \rho' e^{i\omega t}\mathcal{Y}_l^{m_l}, \, ...)$$

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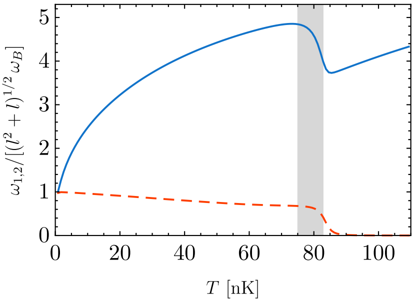
the frequencies ω_1, ω_2 of the hydrodynamic excitations are the main quantitative probe of BKT physics



[AT, Pelster, Salasnich, arXiv:2104.04585]

Thermodynamics

[AT, Salasnich, PRL **123**, 160403 (2019)]: the grand potential reads

$$\Omega = -(4\pi R^2)\frac{\mu^2}{2g_0} + \frac{1}{2}\sum_{l=1}^{l_c}\sum_{m_l=-l}^{l}\left(E_l^B - \epsilon_l - \mu\right) + \frac{1}{\beta}\sum_{l=1}^{\infty}\sum_{m_l=-l}^{l}\ln\left(1 - e^{-\beta E_l^B}\right)$$

with Bogoliubov spectrum $E_l^B = \sqrt{\epsilon_l(\epsilon_l + 2\mu)}$, and $\epsilon_l = \hbar^2 l(l+1)/(2mR^2)$.

Scattering theory calculation on the spherical surface gives

$$g_0 = -\frac{2\pi\hbar^2}{m}\frac{1}{\ln[\sqrt{l_c(l_c+1)}\, a\, e^{\gamma}/(2R)]},$$

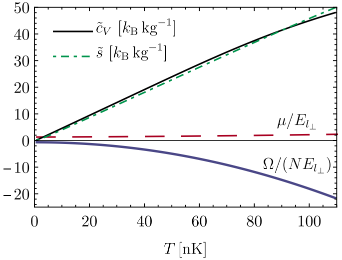
which balances the logarithmic divergence of zero-point energy

Thermodynamics

We obtain the *renormalized* grand potential

$$\frac{\Omega}{4\pi R^2} = -\frac{m\mu^2}{8\pi\hbar^2}\left[\ln\left(\frac{4\hbar^2}{m(E_1^B+\epsilon_1+\mu)a^2\, e^{2\gamma+1}}\right)+\frac{1}{2}\right]$$
$$+\frac{mE_1^B}{8\pi\hbar^2}(E_1^B-\epsilon_1-\mu)+\frac{1}{4\pi R^2}\frac{1}{\beta}\sum_{l=1}^{\infty}\sum_{m_l=-l}^{l}\ln\left(1-e^{-\beta E_l^B}\right),$$

from which we calculate all the thermodynamic functions



While the hydrodynamic excitations are non-monotonic around T_{BKT}, the thermodynamic functions are unaffected by BKT

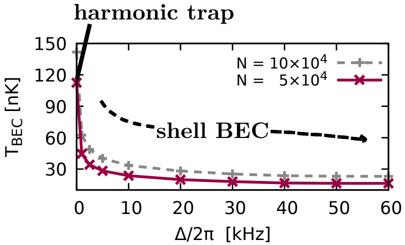
[AT, Pelster, Salasnich, arXiv:2104.04585]

Experimental relevance of finite-temperature properties

Are finite-temperature predictions experimentally relevant?

For the realistic trap parameters of NASA-JPL CAL experiment:

T_{BEC} drops quickly with $\Delta \propto$ shell area



[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

Difficult to reach fully-condensate regime...

$\Rightarrow$ Finite-temperature properties and BKT physics are highly relevant

Conclusion

We assume the vortex-antivortex unbinding as the mechanism of the superfluid transition in shell-shaped condensates,

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- “wavy” imaging pattern
- hydrodynamic modes (vs continuous thermodynamics)

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- hydrodynamic modes (vs continuous thermodynamics)

Finite-size BKT $\leftrightarrow$ curvature of quantum gases

Shell BECs $\leftrightarrow$ platform to study finite-size BKT

Thank you for your attention!

References

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