Quantum Bubbles in Microgravity

Andrea Tononi

Dipartimento di Fisica e Astronomia
"Galileo Galilei", Università di Padova

Work done in collaboration with F. Cinti and L. Salasnich.

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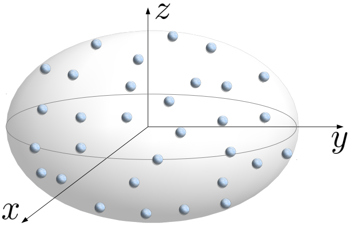
Quantum bubbles

Bose-Einstein condensate on a thin ellipsoidal shell

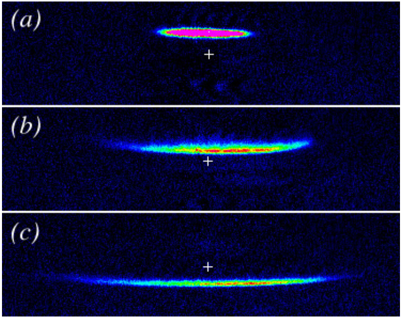


For strong radial confinement:
2D curved superfluid

- finite-size 2D condensate
- topology (and BKT)
- quantized vortices
- free expansion

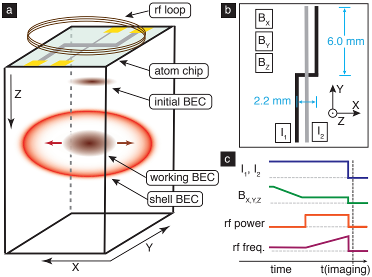
Why microgravity?

Bubble-trap on Earth...



[Colombe, *et al.*, EPL 2004]

NASA-JPL Cold Atom Laboratory

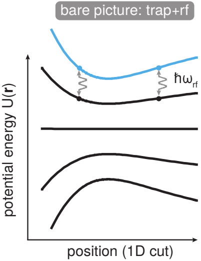


[Lundblad, *et al.*, npj Microgravity 2019]

Outline

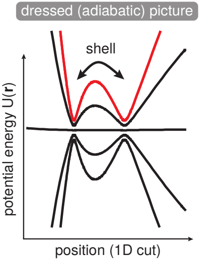
- ▷ Introduction on bubble trapping
- ▷ Bose-Einstein condensation on the surface of a sphere
- ▷ Shell-shaped condensates: statics and dynamics
- ▷ Summary and outlook

Bubble-trapping: experimental realization



Alkali-metal atoms with total angular momentum $F = 2$.

+ Magnetic field $\mathbf{B}(\vec{r}) \implies$ space-dependent Zeeman splitting with $m_F = \{\pm 2, \pm 1, 0\} \implies$ space-dependent bare potentials $u(\vec{r})$



+ Radiofrequency field $\mathbf{B}_{\mathrm{rf}}(\vec{r}, t) \implies$ bubble-trap in the dressed picture (old m_F bad quantum number)

[Lundblad, *et al.*, npj Microgravity 2019]

Bubble-trap

$$U(\vec{r}) = M_F \sqrt{\left[\sum_i m\,\omega_i{}^2 x_i^2/2 - \hbar\Delta\right]^2 + (\hbar\Omega)^2},$$

ω_i: frequencies of the bare harmonic trap

Δ: detuning from the resonant frequency

Ω: Rabi frequency between coupled levels

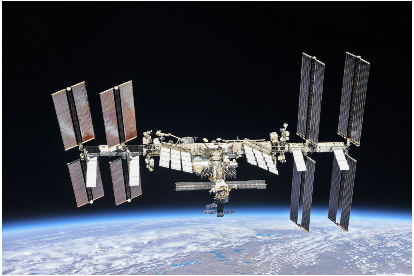
Minimum for $\omega_x{}^2 x^2 + \omega_y{}^2 y^2 + \omega_z{}^2 z^2 = 2\hbar\Delta/m$.

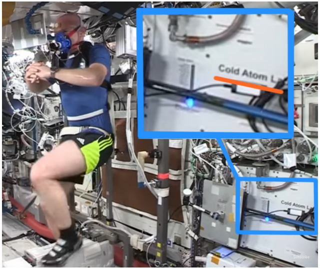
[Zobay, Garraway, PRL 2001]

If gravity is included the **atoms will pool on the bottom of the trap!**

$$U(\vec{r}) = M_F \sqrt{\left[\sum_i m\,\omega_i{}^2 x_i^2/2 - \hbar\Delta\right]^2 + (\hbar\Omega)^2} + \underline{mgz},$$

$\rightarrow$ Experiments on the **Cold Atom Lab** on the International Space Station (PI Nathan Lundblad). [Aveline, *et al.*, Nature **582**, 193 (2020)]





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Bose-Einstein condensation on the surface of a sphere

Noninteracting bosons.

Quantized energy $\varepsilon_l = \frac{\hbar^2}{2mR^2}\, l(l+1)$, with degeneracy $2l+1$.

In the Bose-condensed phase, we can set $\mu = 0$ and

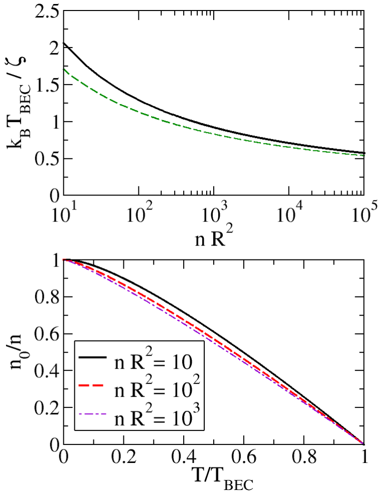
$$N = N_0 + \sum_{l=1}^{+\infty} \frac{2l+1}{e^{\varepsilon_l/(k_B T)} - 1}$$

when $N_0 = 0 \implies T = T_{\mathrm{BEC}}$

BEC on a sphere: **noninteracting case**

$$k_B T_{\text{BEC}} = \frac{\frac{2\pi\hbar^2}{m} n}{\frac{\beta_{\text{BEC}}\hbar^2}{mR^2} - \ln\left(e^{\beta_{\text{BEC}}\hbar^2/mR^2} - 1\right)}$$

$$\frac{n_0}{n} = 1 - \frac{1 - \frac{mR^2}{\hbar^2\beta}\ln\left(e^{\beta\hbar^2/mR^2} - 1\right)}{1 - \frac{mR^2}{\hbar^2\beta_{\text{BEC}}}\ln\left(e^{\beta_{\text{BEC}}\hbar^2/mR^2} - 1\right)}$$



[AT, Salasnich, PRL **123**, 160403 (2019)]

BEC on a sphere: **interacting case**

The grand potential $\Omega = -\beta^{-1} \ln(\mathcal{Z})$, where $\mathcal{Z}$ is the grand canonical partition function

$$\mathcal{Z} = \int \mathcal{D}[\bar{\psi}, \psi] \, e^{-S[\bar{\psi}, \psi]/\hbar},$$

where

$$S[\bar{\psi}, \psi] = \int_0^{\beta\hbar} d\tau \int_0^{2\pi} d\varphi \int_0^{\pi} \sin(\theta) \, d\theta \, R^2 \, \mathcal{L}(\bar{\psi}, \psi)$$

is the Euclidean action, and

$$\mathcal{L} = \bar{\psi}(\theta, \varphi, \tau) \left(\hbar \partial_\tau + \frac{L^2}{2mR^2} - \mu \right) \psi(\theta, \varphi, \tau) + \frac{g}{2} |\psi(\theta, \varphi, \tau)|^4$$

is the Euclidean Lagrangian.

BEC on a sphere: **interacting case**

In the Bose-condensed phase

$$\psi(\theta,\varphi,\tau) = \psi_0 + \eta(\theta,\varphi,\tau)$$

Expanding up to quadratic order, expanding with spherical harmonics, and performing functional integration we get

$$\begin{aligned}\Omega(\mu,\psi_0^2) &= 4\pi R^2\big(-\mu\psi_0^2 + g\psi_0^4/2\big) + \frac{\alpha}{2}\sum_{l=1}^{\infty}\sum_{m_l=-l}^{l} E_l(\mu,\psi_0^2) \\ &+ \frac{\alpha}{\beta}\sum_{l=1}^{\infty}\sum_{m_l=-l}^{l} \ln\Big(1 - e^{-\beta E_l(\mu,\psi_0^2)}\Big) + o(\alpha^2),\end{aligned}$$

with $E_l(\mu,\psi_0^2) = \sqrt{(\epsilon_l - \mu + 2g\psi_0^2)^2 - g^2\psi_0^4}$.

BEC on a sphere: **interacting case**

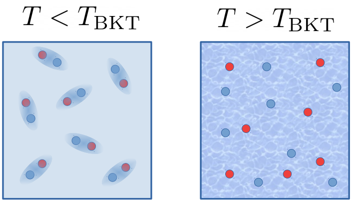
Applying Variational Perturbation Theory we can calculate the critical temperature of the interacting system

$$k_B T_{\rm BEC} = \frac{\frac{2\pi\hbar^2 n}{m} - \frac{gn}{2}}{\frac{\hbar^2\beta_{BEC}}{2mR^2}\left(1+\sqrt{1+\frac{2gmnR^2}{\hbar^2}}\right) - \ln\left(e^{\frac{\hbar^2\beta_{BEC}}{mR^2}\sqrt{1+\frac{2gmnR^2}{\hbar^2}}} - 1\right)}.$$

and the condensate fraction

$$\begin{aligned}\frac{n_0}{n} &= 1 - \frac{mg}{4\pi\hbar^2} - \frac{1}{4\pi R^2 n}\left[1+\sqrt{1+\frac{2gmnR^2}{\hbar^2}}\right] \\ &+ \frac{mk_B T}{2\pi\hbar^2 n}\ln\left(e^{\frac{\hbar^2}{mR^2 k_B T}\sqrt{1+(2gmnR^2/\hbar^2)}} - 1\right).\end{aligned}$$

$R \to \infty$: $T_{\rm BEC} \to 0$, Schick result for quantum depletion.



The unbinding of vortex-antivortex dipoles at $T = T_{\rm BKT}$ destroys the quasi long-range order.

Kosterlitz-Nelson criterion on the sphere [Ovrut, Thomas PRD 1991]

$$k_B T_{BKT} = \frac{\pi}{2} \frac{\hbar^2 n_s(T_{\rm BKT})}{m}$$

with the superfluid density $n_s(T)$ as

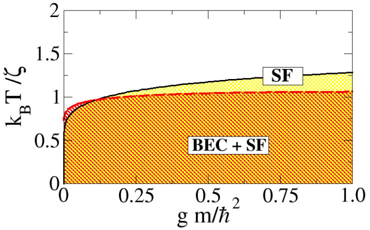
$$n_s = n - \frac{1}{k_B T} \int_1^{+\infty} \frac{dl\,(2l+1)}{4\pi R^2} \frac{\hbar^2 (l^2 + l)}{2mR^2} \frac{e^{E_l^B/(k_B T)}}{(e^{E_l^B/(k_B T)} - 1)^2}.$$

BEC on a sphere: **interacting case**

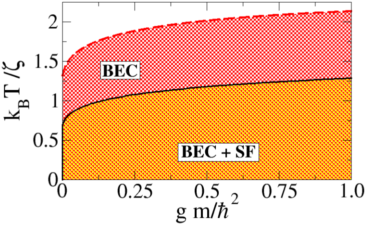
BEC transition (red dashed)
BKT=SF transition (**black**)

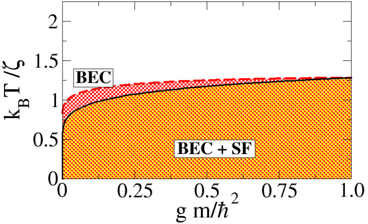
Plots: $nR^2 =$ 10^2
10^5 10^4

Usual 2D picture (thermodyn. limit)



Region of BEC only





[AT, Salasnich, PRL **123**, 160403 (2019)]

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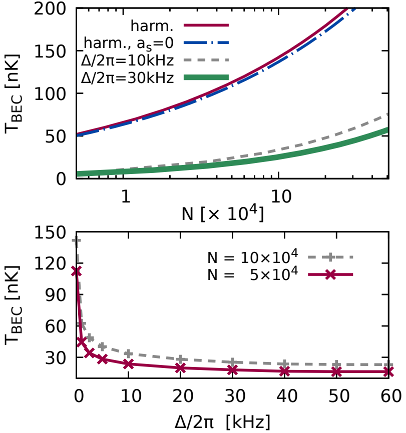
Shell-shaped (ellipsoidal) condensates

For the realistic trap parameters of NASA-JPL CAL experiment:

$$T_{BEC}^{bubble\ trap} \ll T_{BEC}^{harmonic\ trap}$$

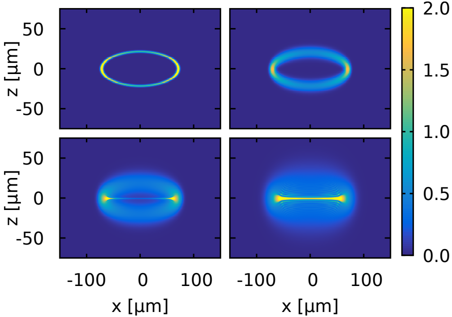


[AT, Cinti, Salasnich, arxiv:1912.07297]

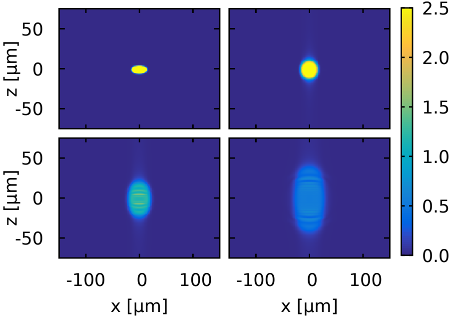
(using Hartree-Fock theory [Giorgini, *et al.*, J. Low T. Phys., **109**, 309 (1997)])

Free expansion

Bubble trap



Harmonic trap



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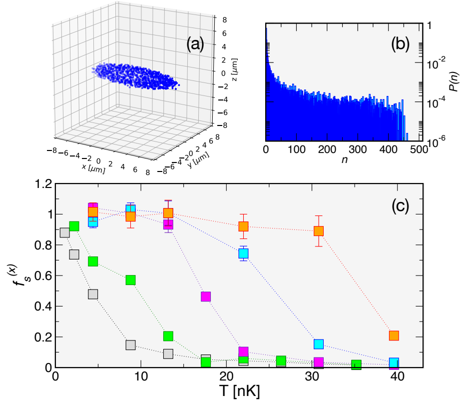
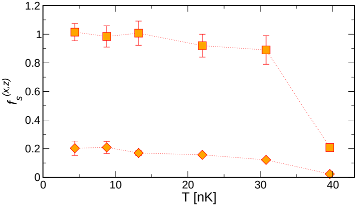
Path Integral Monte Carlo simulations





anisotropic $f_s^{(x,z)}$ from nonclassical moment of inertia

[AT, Cinti, Salasnich, arxiv:1912.07297]

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Summary and outlook

SPHERICAL TRAP

- ◇ Calculation of $T_{\rm BEC}$, $T_{\rm BKT}$, and n_0/n for spherical trap

ELLIPSOIDAL TRAP

- ◇ $T_{\rm BEC}$ and realistic experimental description
- ◇ Free expansion
- ◇ Superfluidity studied with Quantum Monte Carlo

Future projects

Vortices on a compact superfluid

Extension of the analytical calculations

Quantum bubbles in microgravity

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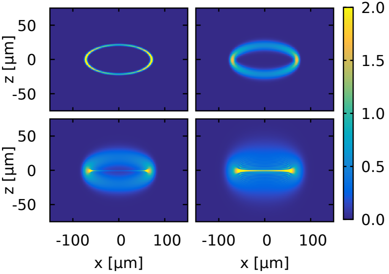

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References

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