Hydrodynamic excitations in bosonic and fermionic 2D superfluids

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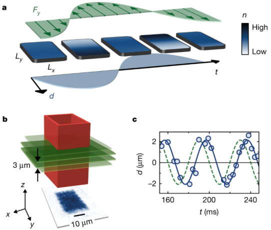
Seminar at Quantech group (UFRN) – 02/09/2021



sound propagation in a 2D bosonic superfluid, from [Christodoulou, et al. Nature **594**, 191 (2021)]

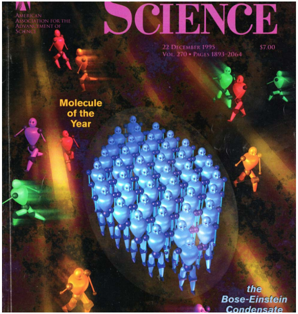
Collaborators: Bighin, Cappellaro, Cinti, Furutani, Pelster, Salasnich

This presentation is on www.andreatononi.com

Outline

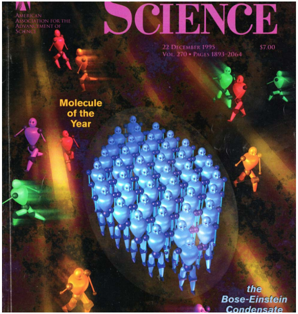
- Ultracold Atomic Gases
  - Bose-Einstein condensation
  - Superfluidity and the BKT transition
- Landau two-fluid model
- Hydrodynamic excitations in 2D superfluids
  - Bosonic superfluids
  - Fermionic superfluids
  - Shell-shaped superfluids
- Conclusions

Bose-Einstein condensation



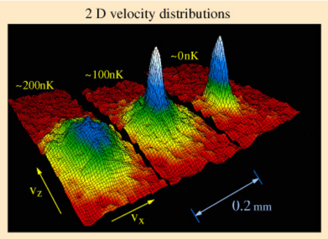
Bose-Einstein condensate:
a many-body system of identical bosonic particles of which a **macroscopic fraction** occupies the same lowest-energy **single-particle state**

Bose-Einstein condensation

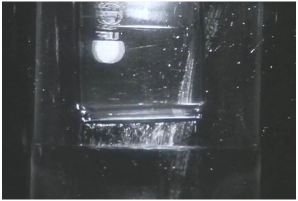


Bose-Einstein condensate:
a many-body system of identical bosonic particles of which a **macroscopic fraction** occupies the same lowest-energy **single-particle state**

In 1995 (Cornell & Wieman, Ketterle):
Bose-Einstein condensation **observed experimentally** in ^{87}Rb and ^{23}Na gases through laser cooling and evaporative cooling

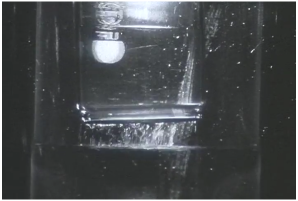


Superfluidity



Superfluidity:
frictionless flow of a quantum liquid through narrow capillaries

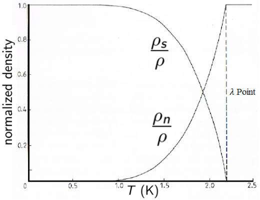
Superfluidity



Superfluidity:
frictionless flow of a quantum liquid through narrow capillaries

Kapitza, in 1938:
observation of **superfluidity** in liquid ^4He below $T_\lambda = 2.17\,K$

Landau & Tisza, in 1941:
two-fluid model



What is the relation between Bose-Einstein condensation and superfluidity?

Bose-Einstein condensation

quantum statistical phenomenon

Superfluidity

transport phenomenon

BEC and superfluidity

For weakly-interacting bosons:

$3D$ $T_{BEC} = T_{\text{superfluidity}} \sim T_{BEC}^{(0)} = \frac{2\pi\hbar^2}{mk_B}\frac{n^{2/3}}{\zeta(3/2)^{2/3}}$

$2D$ $T_{BEC} = 0$: no long-range order at finite temperature...

...“*Hohenberg-Mermin-Wagner theorem*”:
no BEC at finite temperature in the thermodynamic limit for
$D = 1, 2$

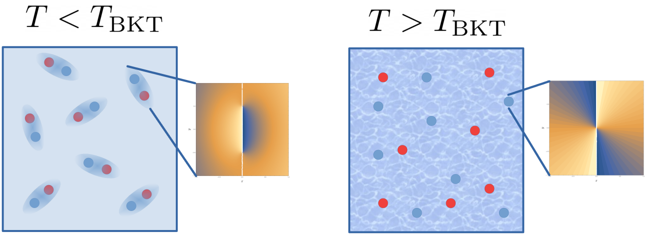
[Hohenberg, PR **158**, 383 (1967)] [Mermin, Wagner, PRL **17**, 1133 (1966)]

...but superfluidity (“quasi-long-range order”) at $T < \mathbf{T_{BKT}}$

($\mathbf{T_{BKT}}$: Berezinskii-Kosterlitz-Thouless transition temperature)

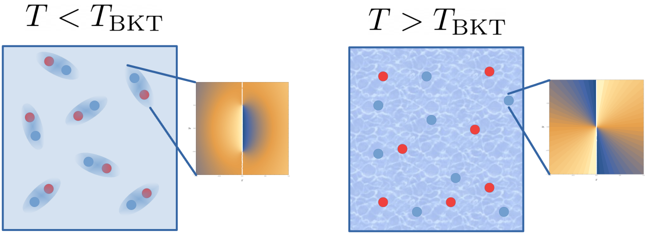
Berezinskii-Kosterlitz-Thouless transition

Vortex-antivortex dipoles at $T < T_{\mathrm{BKT}}$, free vortices at $T > T_{\mathrm{BKT}}$



Berezinskii-Kosterlitz-Thouless transition

Vortex-antivortex dipoles at $T < T_{\rm BKT}$, free vortices at $T > T_{\rm BKT}$



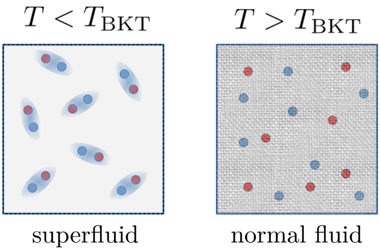
Simplest calculation of T_{BKT}:

Free energy of a vortex in a 2D infinite superfluid:

$$F = U - TS = \frac{\pi\hbar^2 n_s^{(0)}(T)}{m} \ln\left(\frac{L}{\xi}\right) - T\, k_B \ln\left(\frac{L^2}{\xi^2}\right)$$

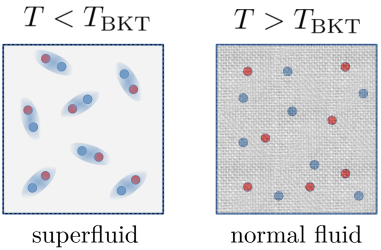
Vortex appears when $F < 0$, namely $T > T_{\rm BKT} = \frac{\pi\hbar^2 n_s^{(0)}(T)}{2mk_B}$

Berezinskii-Kosterlitz-Thouless transition



How to go beyond the single-vortex calculation?

Berezinskii-Kosterlitz-Thouless transition



How to go beyond the single-vortex calculation?

Adimensional parameters

$K(\ell) = \frac{\hbar^2 n_s(\ell)}{m k_B T}$; $y(\ell) \sim e^{-\beta \mu_v(\ell)}$

RG scale $\ell = \ln(r/\xi)$,

Distance between vortices:

$r \in [\xi, \infty]$

[Nelson, Kosterlitz, PRL **39**, 1201 (1977)]

RG equations of a flat superfluid

$$\frac{dK^{-1}(\ell)}{d\ell} = -4\pi^3 y^2(\ell)$$

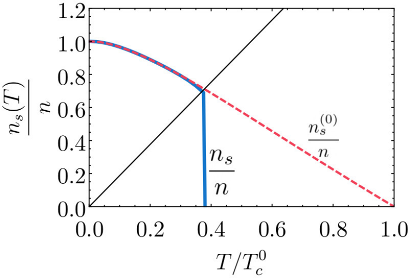
$$\frac{dy(\ell)}{d\ell} = [2 - \pi K(\ell)]\, y(\ell)$$

$\rightarrow$ From bare $n_s(\ell = 0) = n_s^{(0)}$ to renormalized $n_s = n_s(\ell = \infty)$

Berezinskii-Kosterlitz-Thouless transition

Universal jump of the superfluid density at the

Kosterlitz-Nelson criterion: $\frac{n_s(T_{\mathrm{BKT}}^-)}{T_{\mathrm{BKT}}} = \frac{2}{\pi}\frac{mk_B}{\hbar^2}$



[Nelson, Kosterlitz, PRL **39**, 1201 (1977)]

Goal of today's talk:

theoretical and experimental analysis of the **hydrodynamic excitations in 2D superfluids**, which conjugate BKT physics (superfluidity) and thermodynamics (Bose-Einstein condensation)

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Landau two-fluid model

Phenomenological description of a quantum liquid as composed by

- Superfluid: zero viscosity, no entropy
- Normal fluid: viscous, carries all the system entropy

[Landau J. Phys. (USSR) **5**, 71 (1941)]

Landau two-fluid model

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- Normal fluid: viscous, carries all the system entropy

Total mass density:

$$\rho = \rho_s + \rho_n$$

Mass current:

$$\mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$$

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Landau two-fluid model

Phenomenological description of a quantum liquid as composed by

- Superfluid: zero viscosity, no entropy
- Normal fluid: viscous, carries all the system entropy

Hydrodynamic equations (linearized):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

$$\frac{\partial \rho \tilde{s}}{\partial t} + \rho \tilde{s} \, \nabla \cdot \mathbf{v}_n = 0$$

$$\frac{\partial \mathbf{j}}{\partial t} + \nabla P = 0$$

$$m \frac{\partial \mathbf{v}_s}{\partial t} + \nabla \mu = 0$$

Total mass density:

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Mass current:

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Hydrodynamic modes (sound waves)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

$$\frac{\partial \rho \tilde{s}}{\partial t} + \rho \tilde{s} \, \nabla \cdot \mathbf{v}_n = 0$$

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$$m \frac{\partial \mathbf{v}_s}{\partial t} + \nabla \mu = 0$$

two coupled sound equations

(III $\rightarrow \partial_t$I):

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 P$$

(I $\rightarrow$ III$, \rho, \ldots$):

$$\frac{\partial^2 \tilde{s}}{\partial t^2} = \tilde{s}^2 \frac{\rho_s}{\rho_n} \nabla^2 T$$

[Landau J. Phys. (USSR) **5**, 71 (1941)]

Hydrodynamic modes (sound waves)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

$$\frac{\partial \rho \tilde{s}}{\partial t} + \rho \tilde{s}\, \nabla \cdot \mathbf{v}_n = 0$$

$$\frac{\partial \mathbf{j}}{\partial t} + \nabla P = 0$$

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two coupled sound equations

$(\mathrm{III} \to \partial_t \mathrm{I})$:

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 P$$

$(\mathrm{I} \to \mathrm{III}, \rho, \ldots)$:

$$\frac{\partial^2 \tilde{s}}{\partial t^2} = \tilde{s}^2 \frac{\rho_s}{\rho_n} \nabla^2 T$$

Fluctuations around the equilibrium configuration:

$$\rho \sim \rho_0 + \left(\tfrac{\partial \rho}{\partial P}\right)_T \delta P(\omega)\, e^{i\omega(t - x/c)} + \left(\tfrac{\partial \rho}{\partial T}\right)_P \delta T(\omega)\, e^{i\omega(t - x/c)},$$

$$\tilde{s} \sim \tilde{s}_0 + \left(\tfrac{\partial \tilde{s}}{\partial P}\right)_T \delta P(\omega)\, e^{i\omega(t - x/c)} + \left(\tfrac{\partial \tilde{s}}{\partial T}\right)_P \delta T(\omega)\, e^{i\omega(t - x/c)}$$

[Landau J. Phys. (USSR) **5**, 71 (1941)]

Hydrodynamic modes (sound waves)

$$\begin{cases} \delta P(\omega)\left[-c^2\left(\frac{\partial\rho}{\partial P}\right)_T+1\right]+\delta T(\omega)\left[-c^2\left(\frac{\partial\rho}{\partial T}\right)_P\right]=0, \\ \delta P(\omega)\left[-c^2\left(\frac{\partial\tilde{s}}{\partial P}\right)_T\right]+\delta T(\omega)\left[-c^2\left(\frac{\partial\tilde{s}}{\partial T}\right)_P+\tilde{s}^2\frac{\rho_s}{\rho_n}\right]=0, \end{cases}$$

and setting $\det = 0$ we get the biquadratic equation:

$$c^4 - c^2\left[\left(\frac{\partial P}{\partial\rho}\right)_{\tilde{s}} + \frac{T\tilde{s}^2\rho_s}{\tilde{c}_V\rho_n}\right] + \frac{\rho_s T\tilde{s}^2}{\rho_n\tilde{c}_V}\left(\frac{\partial P}{\partial\rho}\right)_T = 0$$

...Landau two-fluid model predicts two sound velocities:

$$c_{1,2} = \left[\frac{v_A^2+v_L^2}{2} \pm \sqrt{\left(\frac{v_A^2+v_L^2}{2}\right)^2 - v_L^2 v_T^2}\right]^{1/2}$$

$$v_A = \sqrt{\left(\frac{\partial P}{\partial\rho}\right)_{\tilde{s}}}, \quad v_T = \sqrt{\left(\frac{\partial P}{\partial\rho}\right)_T}, \quad v_L = \sqrt{\frac{\rho_s T\tilde{s}^2}{\rho_n\tilde{c}_V}}$$

(adiabatic, isothermal, Landau velocities)

Hydrodynamic modes (sound waves)

$$c_{1,2} = \left[\frac{v_A^2 + v_L^2}{2} \pm \sqrt{\left(\frac{v_A^2 + v_L^2}{2} \right)^2 - v_L^2 v_T^2} \right]^{1/2}$$

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The sound velocities are determined by:

— thermodynamics

— superfluid density

[Landau J. Phys. (USSR) **5**, 71 (1941)]

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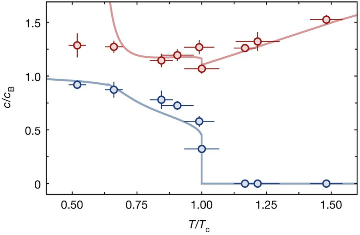
Sound propagation in 2D bosonic superfluids: experiment

Excitation of sounds across the BKT transition with a time-dependent magnetic potential



($T_c^{exp} = 42\,\mathrm{nK}$ experimentally)

$$c_{1,2} = \left[\frac{v_A^2 + v_L^2}{2} \pm \sqrt{\left(\frac{v_A^2 + v_L^2}{2}\right)^2 - v_L^2 v_T^2}\right]^{1/2}$$

$$v_{\{A,T\}} = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_{\{\tilde{s},T\}}}, \quad v_L = \sqrt{\frac{\rho_s T \tilde{s}^2}{\rho_n \tilde{c}_V}}$$

At the BKT transition $c_2 \to 0$, while $c_1 \to v_A$

[Christodoulou, et al. Nature **594**, 191 (2021)]

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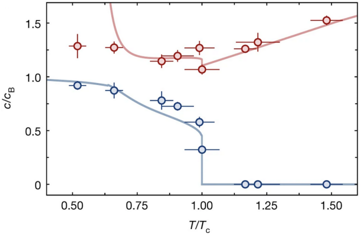
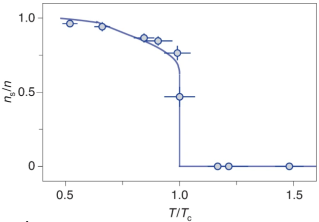


($T_c^{exp} = 42\,\text{nK}$ experimentally)



(theory for thermodynamics with $T_c^{th} = 37\,\text{nK}$)
$\Rightarrow$ calculation of n_s/n

At the BKT transition $c_2 \to 0$, while $c_1 \to v_A$

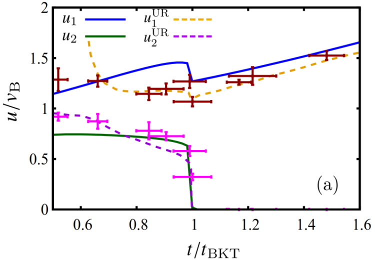
[Christodoulou, et al. Nature **594**, 191 (2021)]

Sound propagation in 2D bosonic superfluids: our theory

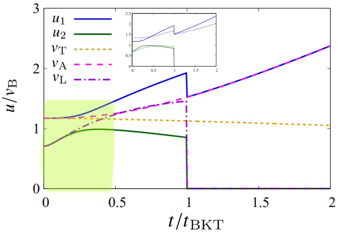
Thermodynamics: derived from the free energy

$$F = \frac{g}{2}\frac{N^2}{L^D} + \frac{1}{2}\sum_{\mathbf{p}} E_p + \frac{1}{\beta}\sum_{\mathbf{p}} \ln\left[1 - e^{-\beta E_p}\right], \quad E_p = \sqrt{\frac{p^2}{2m}\left(\frac{p^2}{2m} + 2gn\right)},$$

superfluid density ρ_s: solve RG equations up to finite system size



good agreement



predictions for low T, low g!

...and we obtain more results also in 3D and in 1D

→ [Furutani, AT, Salasnich, New J. Phys. **23**, 043043 (2021)]

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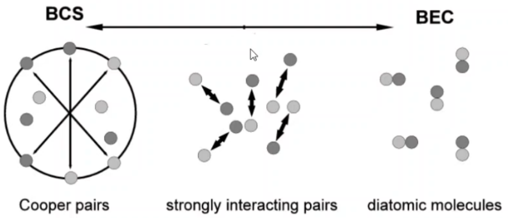
Fermionic superfluids: BCS-BEC crossover

We analyze 2D fermions with *attractive* interactions, with Hamiltonian

$$\hat{H} = \sum_{\sigma=\uparrow,\downarrow} \int_{L^2} \mathrm{d}^2\mathbf{r} \left\{ \hat{\psi}^\dagger_{\sigma,\mathbf{r}} \left(-\frac{\hbar^2 \nabla^2}{2m} \right) \hat{\psi}_{\sigma,\mathbf{r}} + g \hat{\psi}^\dagger_{\uparrow,\mathbf{r}} \hat{\psi}^\dagger_{\downarrow,\mathbf{r}} \hat{\psi}_{\downarrow,\mathbf{r}} \hat{\psi}_{\uparrow,\mathbf{r}} \right\}$$

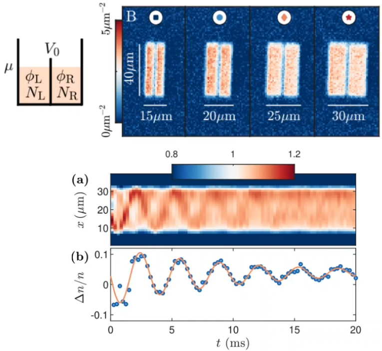
...tuning g one realizes the whole BCS-BEC crossover



Pairing gap

$$\begin{aligned} \hat{\Delta}_\mathbf{r} &= g \hat{\psi}_{\downarrow,\mathbf{r}} \hat{\psi}_{\uparrow,\mathbf{r}} \\ &= \Delta_0 + \hat{\eta}_\mathbf{r} \end{aligned}$$

Sound propagation in 2D fermionic superfluids: experiment

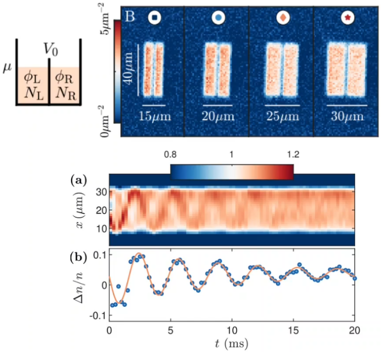


Excitation protocol: phase imprinting on one half of the system...

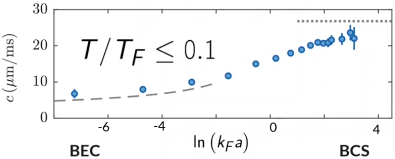
[Luick, et al., Science **369**, 89 (2020)] [Bohlen, et al., PRL **124**, 240403 (2020)]

Sound propagation in 2D fermionic superfluids: experiment

Excitation protocol: phase imprinting on one half of the system...



...seems to excite **only one sound mode**!



[Luick, et al., Science **369**, 89 (2020)] [Bohlen, et al., PRL **124**, 240403 (2020)]

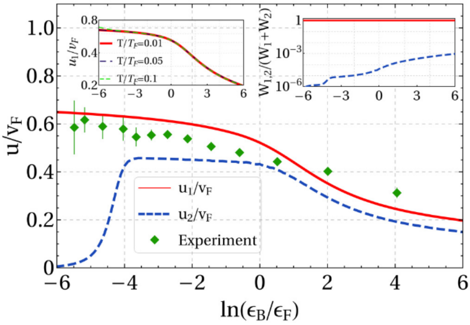
Sound propagation in 2D fermionic superfluids: our theory

Thermodynamics from the grand potential

$$\Omega = \frac{1}{\beta}\sum_{\mathbf{k}}\left(\ln\{2\cosh[\beta E_{sp}(k)]\} - \frac{\hbar^2 k^2}{2m} + \mu\right) - L^2\frac{\Delta_0^2}{g} + \frac{1}{2\beta}\sum_{\mathbf{Q}}\ln\det\mathbb{M}(Q),$$

$$E_{sp}(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m} - \mu\right)^2 + \Delta_0^2}, \qquad \det\mathbb{M}(\mathbf{q},\omega) = 0 \to \hbar\omega_{col}(\mathbf{q})$$

+ superfluid density by solving the RG equations... sounds:



Only the first sound is excited in this experiment!

[A. Tononi, et al. Phys. Rev. A **103**, L061303 (2021)]

Sound propagation in 2D fermionic superfluids: our theory

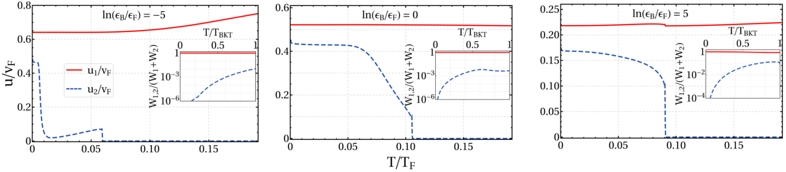
A density perturbation does not excite evenly first and second sound, but with different weights

$$\delta\rho(\mathbf{r},t) = W_1\,\delta\rho_1(\mathbf{r}\pm c_1 t, t) + W_2\,\delta\rho_2(\mathbf{r}\pm c_2 t, t),$$

$$\frac{W_1}{W_1+W_2} = \frac{(c_1^2 - v_L^2)\,c_2^2}{(c_1^2 - c_2^2)\,v_L^2}, \quad \frac{W_2}{W_1+W_2} = \frac{(v_L^2 - c_2^2)\,u_1^2}{(c_1^2 - c_2^2)\,v_L^2}$$

in this setup $c_2 \approx v_L \Rightarrow$ only the first sound is excited.

$\Rightarrow$ a heat probe excites mainly the second sound (still unobserved in uniform fermions), and we offer finite-temperature predictions



[A. Tononi, et al. Phys. Rev. A **103**, L061303 (2021)]

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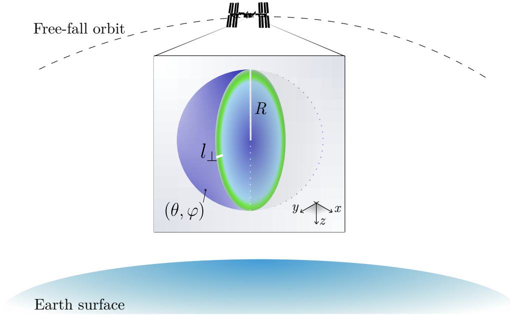
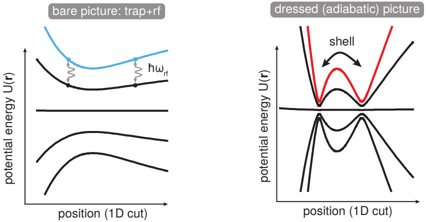
Shell-shaped superfluids: what are they?



In short: a weakly-interacting two-dimensional Bose gas on the surface of a sphere, see [AT, Salasnich, PRL **123**, 160403 (2019)]

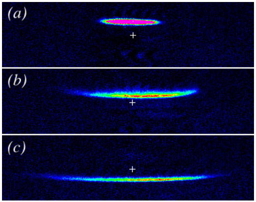
Shell-shaped superfluids

Bubble-trap…



[Lundblad *et al.*, npj Microgravity **5**, 30 (2019)]

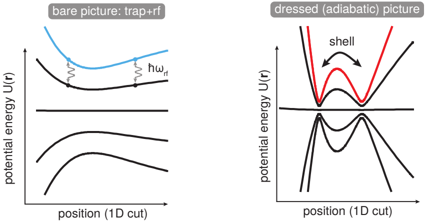
…on Earth



[Colombe *et al.*, EPL **67**, 593 (2004)]

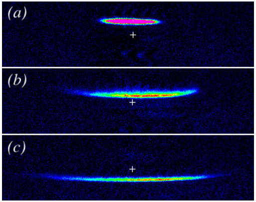
Shell-shaped superfluids

Bubble-trap...



[Lundblad *et al.*, npj Microgravity **5**, 30 (2019)]

...on Earth

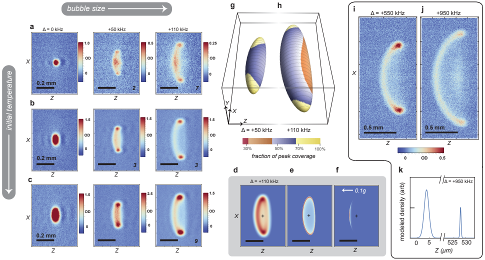


[Colombe *et al.*, EPL **67**, 593 (2004)]

⇒ Experiments on NASA **Cold Atom Lab**

[Carollo *et al.*, arXiv:2108.05880]

[Aveline *et al.*, Nature **582**, 193 (2020)]

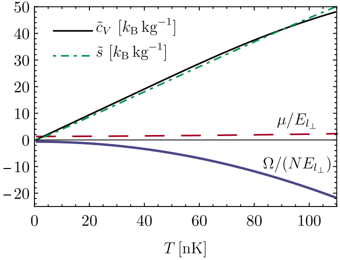


Shell-shaped superfluids: thermodynamics

Starting from [AT, Salasnich, PRL **123**, 160403 (2019)], we calculate the grand potential:

$$\frac{\Omega}{4\pi R^2} = -\frac{m\mu^2}{8\pi\hbar^2}\left[\ln\left(\frac{4\hbar^2}{m(E_1^B+\epsilon_1+\mu)a^2\,e^{2\gamma+1}}\right)+\frac{1}{2}\right]$$
$$+\frac{mE_1^B}{8\pi\hbar^2}(E_1^B-\epsilon_1-\mu)+\frac{1}{4\pi R^2}\frac{1}{\beta}\sum_{l=1}^{\infty}\sum_{m_l=-l}^{l}\ln\left(1-e^{-\beta E_l^B}\right),$$

from which we derive all the thermodynamic functions



[AT, Pelster, Salasnich, arXiv:2104.04585]

Shell-shaped superfluids: BKT transition

RG equations of a spherical superfluid

$$\frac{dK^{-1}(\theta)}{d\ell(\theta)} = -4\pi^3 y^2(\theta)$$

$$\frac{dy(\theta)}{d\ell(\theta)} = [2 - \pi K(\theta)]\, y(\theta)$$

RG scale?

$$\ell(\theta) = \ln[2R\sin(\theta/2)/\xi]$$

Distance between vortices:

$2R\sin(\theta/2) \in [\xi, 2R]$...

...but in 3D space!!

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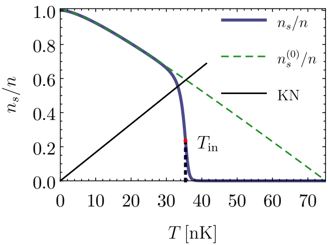
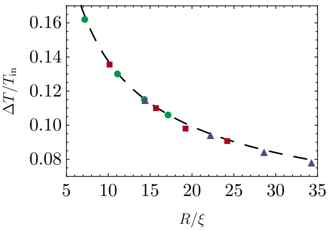
Finite system size $\Rightarrow$ **smooth** vanishing of n_s



$\Delta T/T_{\rm in} \propto \ln^{-2}(R/\xi)$



[AT, Pelster, Salasnich, arXiv:2104.04585]

Hydrodynamic modes in shell-shaped superfluids

Thermodynamics and superfluid density

$\Rightarrow$ calculate the sound velocities...

[AT, Pelster, Salasnich, arXiv:2104.04585]

Hydrodynamic modes in shell-shaped superfluids

Thermodynamics and superfluid density

$\Rightarrow$ calculate the sound velocities... **Sound**??

The correct basis is spherical harmonics $\mathcal{Y}_l^{m_l}$, not plane waves:

$$\left(\rho \sim \rho_0 + (\tfrac{\partial \rho}{\partial P})_T \, \delta P(\omega) \, e^{i\omega t} \mathcal{Y}_l^{m_l} + (\tfrac{\partial \rho}{\partial T})_P \, \delta T(\omega) \, e^{i\omega t} \mathcal{Y}_l^{m_l}, \, \ldots\right)$$

$$\omega_{1,2}^2 = \left[\frac{l(l+1)}{R^2}\right] \left[\frac{v_A^2 + v_L^2}{2} \pm \sqrt{\left(\frac{v_A^2 + v_L^2}{2}\right)^2 - v_L^2 v_T^2}\right]$$

[AT, Pelster, Salasnich, arXiv:2104.04585]

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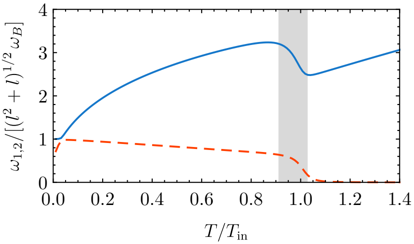
the frequencies ω_1, ω_2 of the hydrodynamic excitations are the main quantitative probe of BKT physics



[AT, Pelster, Salasnich, arXiv:2104.04585]

Outline

- Ultracold Atomic Gases
  - Bose-Einstein condensation
  - Superfluidity and the BKT transition
- Landau two-fluid model
- Hydrodynamic excitations in 2D superfluids
  - Bosonic superfluids
  - Fermionic superfluids
  - Shell-shaped superfluids
- Conclusions

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In 2D, the hydrodynamic excitations offer a direct evidence of the BKT transition and of the system thermodynamics.

Conclusions

The recent experiments with *bosonic* and *fermionic* gases confirm that the Landau two-fluid model is a valid description of weakly-interacting superfluids.

In 2D, the hydrodynamic excitations offer a direct evidence of the BKT transition and of the system thermodynamics.

Driving future experiments:

- second sound in 2D uniform fermions
- BKT physics in shell-shaped superfluids

Thank you for your attention!

References

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