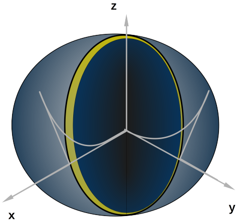
The physics of shell-shaped Bose-Einstein condensates



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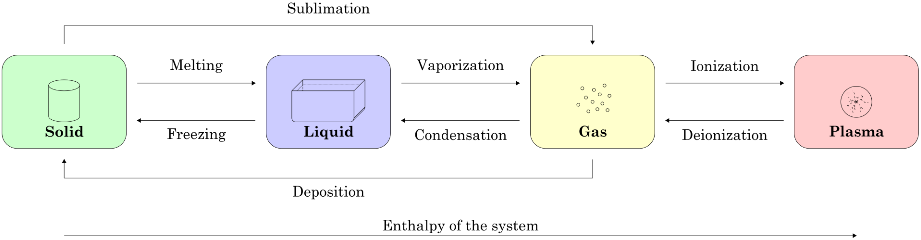
This presentation on www.andreatononi.com

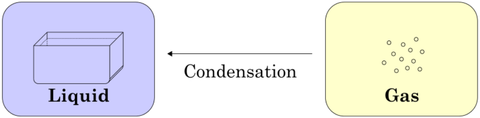
Outline

- Bose-Einstein Condensation and Superfluidity
- The physics of shell-shaped BECs
  - Properties and challenges of shell-shaped condensates
  - Bose-Einstein condensation on the surface of a sphere
  - Recent developments
- Conclusions

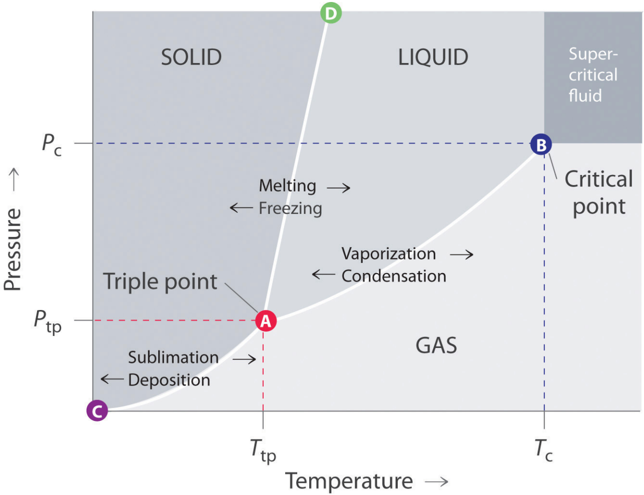
Phases of matter



Condensation

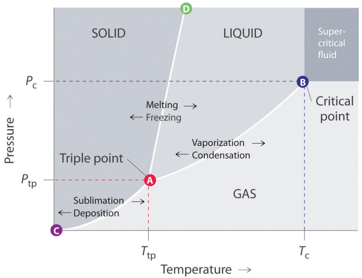


Phases of matter at equilibrium



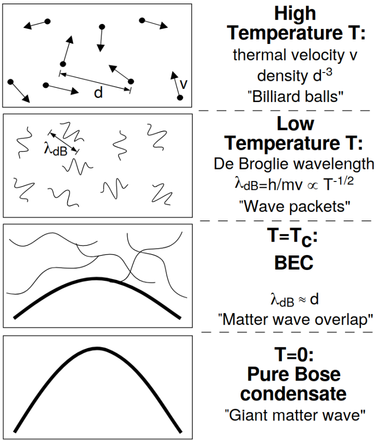
A metastable phase of matter: Bose-Einstein condensate

Cooling a gas at very low pressure and density: **solid** at equilibrium...

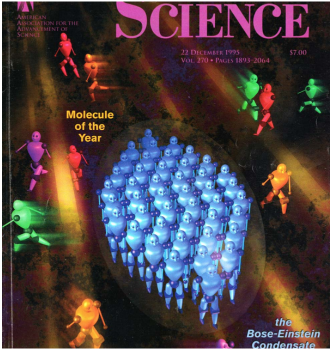


...but (in a bosonic system) a Bose-Einstein condensate as a metastable state!

How a BEC forms:

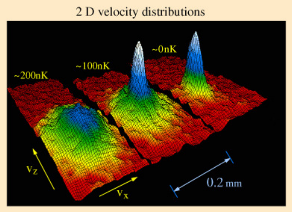


Bose-Einstein Condensation

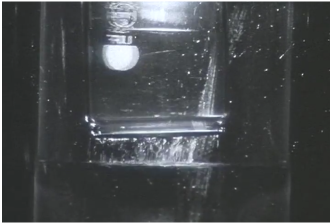


Bose-Einstein condensate:
a many-body system of identical bosonic particles of which a **macroscopic fraction** occupies the same lowest-energy **single-particle state**

Cornell & Wieman, Ketterle, in 1995:
observation of **Bose-Einstein condensation** of ^{87}Rb and ^{23}Na gases through laser cooling and evaporative cooling



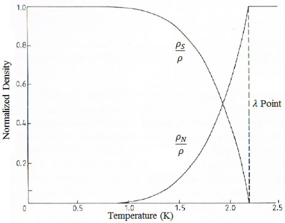
Superfluidity



Superfluidity:
frictionless flow of a quantum liquid through narrow capillaries

Kapitza, in 1938:
observation of **superfluidity** in liquid ^{4}He below $T_\lambda = 2.17\,\mathrm{K}$

Landau & Tisza, in 1941:
two-fluid model



BEC and Superfluidity of noninteracting bosons

No superfluidity!

What about BEC?

$3D \quad T_{BEC}^{(0)} = \frac{2\pi\hbar^2}{mk_B}\frac{n^{2/3}}{\zeta(3/2)^{2/3}}$

$2D \quad T_{BEC}^{(0)} = 0$: no long-range order...

...“*Hohenberg-Mermin-Wagner theorem*”:
no BEC at finite temperature in the thermodynamic limit for $D = 1, 2$
[Hohenberg, PR **158**, 383 (1967)] [Mermin, Wagner, PRL **17**, 1133 (1966)]

BEC and Superfluidity of **interacting** bosons

For weakly-interacting bosons:

$3D \;\; T_{BEC} = T_{\text{superfluidity}} \sim T_{BEC}^{(0)}$

$2D \;\; T_{BEC} = 0$: no long-range order...

...but superfluidity ("quasi-long-range order") at $T < \mathbf{T_{BKT}}$

Berezinskii-Kosterlitz-Thouless transition

Quasi-order parameter of a 2D superfluid:

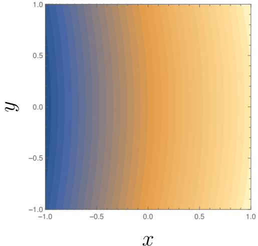
$$\psi(\vec{r}) = [n_s^{(0)}(T)]^{1/2}\, e^{i\theta(\vec{r})}$$

The kinetic energy includes only phase fluctuations:

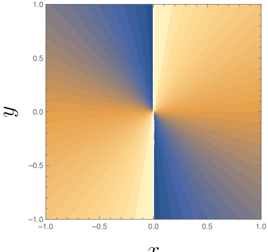
$$U = \int d^2r\, \psi^*(\vec{r})\big(-\frac{\hbar^2\nabla^2}{2m}\big)\psi(\vec{r}) = \frac{\hbar^2 n_s^{(0)}}{2m}\int d^2r\,(\nabla\theta)^2$$

$\theta(\vec{r})$ invariant under $\theta(\vec{r}) \to \theta(\vec{r}) + 2\pi q, \quad q \in \mathcal{Z}$

$\theta(\vec{r}) \sim const$



$\theta(\vec{r}) = q\arctan(y/x)$



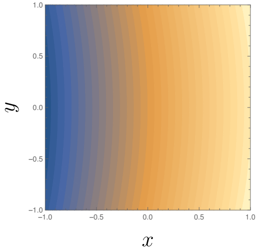
Berezinskii-Kosterlitz-Thouless transition

Introducing the superfluid velocity $\vec{v}_s = \frac{\hbar}{m}\nabla\theta(\vec{r})$,

which satisfies $\frac{m}{\hbar}\oint_\gamma \vec{v}_s \cdot d\vec{l} = 2\pi q$ (Feynman-Onsager) ,

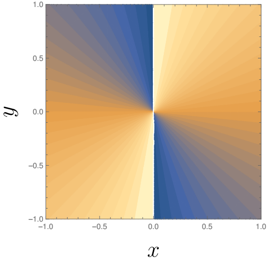
the energy reads $U = \frac{1}{2} m n_s^{(0)} \int d^2 r \, v_s^2$

$\theta(\vec{r}) \sim const$



low-energy configuration

$\theta(\vec{r}) = q \arctan(y/x)$

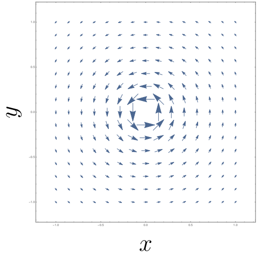


high-energy **topological** configuration

Topological configurations?

A vortex is a **topological** global property of the system.

The velocity circulation $\frac{m}{\hbar}\oint_\gamma \vec{v}_s \cdot d\vec{l} = 2\pi q$, **does not depend** on the specific circuit γ considered, which can be deformed continuously ($\leftrightarrow$ topology) around the vortex core.



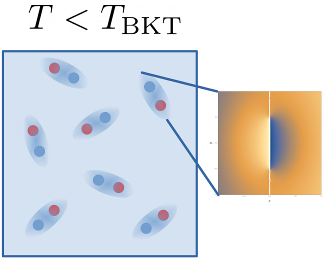
vortex velocity field

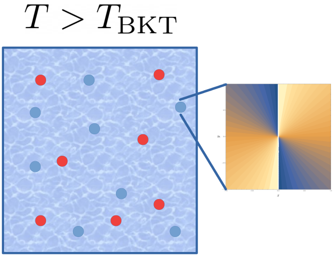
“whorl fingerprint”

Berezinskii-Kosterlitz-Thouless transition

Free energy of a vortex: $F = U - TS = \frac{\pi\hbar^2 n_s^{(0)}(T)}{m} \ln\left(\frac{L}{a}\right) - T\, k_B \ln\left(\frac{L^2}{a^2}\right)$

Vortices appear when $F < 0$, namely $T > T_{\mathrm{BKT}} = \frac{\pi\hbar^2 n_s^{(0)}(T)}{2mk_B}$



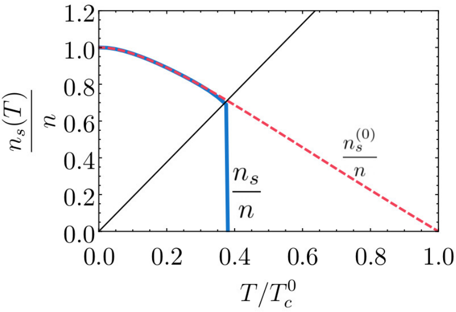


Vortex-antivortex dipoles at $T < T_{\mathrm{BKT}}$, free vortices at $T > T_{\mathrm{BKT}}$

Berezinskii-Kosterlitz-Thouless transition

[Nelson, Kosterlitz, PRL **39**, 1201 (1977)]

Renormalization group equations to calculate the renormalized superfluid density, $n_s(T)$, using the bare superfluid density, $n_s^{(0)}$, as initial condition.



Kosterlitz-Nelson criterion: $\frac{n_s(T_{\text{BKT}}^-)}{T_{\text{BKT}}} = \frac{2}{\pi}\frac{mk_B}{\hbar^2}$

The Nobel Prize in Physics 2016

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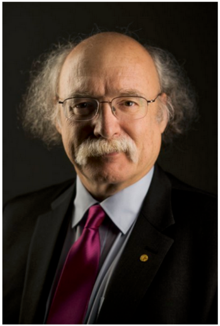
David J. Thouless

Prize share: 1/2



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J. Michael Kosterlitz

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"for theoretical discoveries of topological phase transitions and topological phases of matter"

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- Bose-Einstein Condensation and Superfluidity
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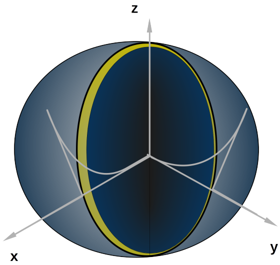
Shell-shaped Bose-Einstein condensate



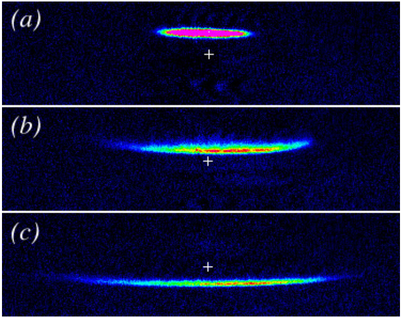
BEC on the surface of an ellipsoid

Why should we care?

- BEC in 2D (finite size)
- curved quantum system
- BKT, topology, vortices
- experimentally realizable

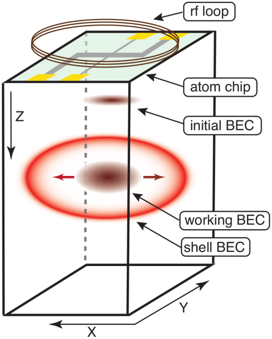
The experiments with "bubble" traps

technically difficult on Earth...



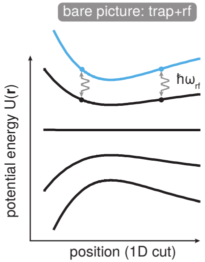
[Colombe *et al.*, EPL **67**, 593 (2004)]

NASA-JPL Cold Atom Laboratory



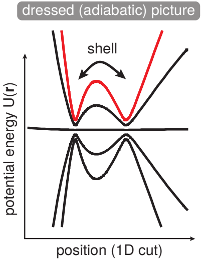
[Lundblad *et al.*, npj Microgravity **5**, 30 (2019)]

How to implement bubble traps



Alkali-metal atoms
(here: total angular momentum $F = 2$)

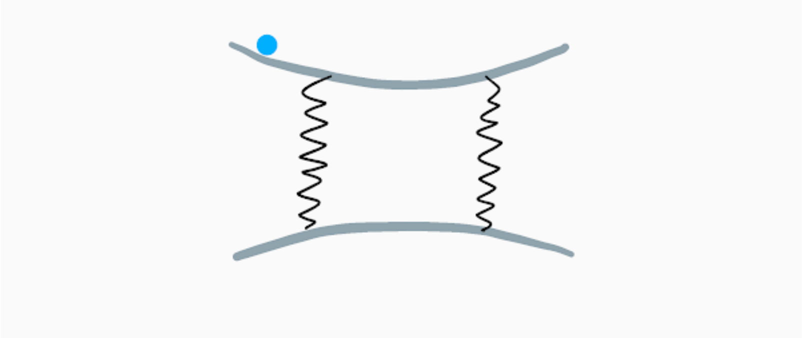
$+$ Magnetic field $\mathbf{B}(\vec{r})$
$\Longrightarrow$ space-dependent Zeeman splitting with $m_F = \{\pm 2, \pm 1, 0\}$ $\Longrightarrow$ space-dependent bare potentials $u(\vec{r})$



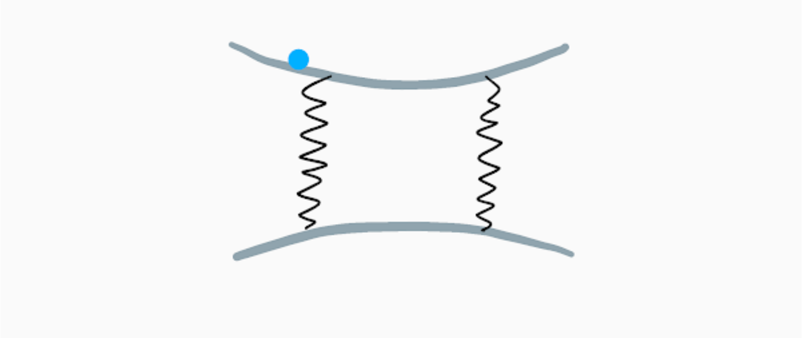
$+$ Radiofrequency field $\mathbf{B}_{\mathrm{rf}}(\vec{r}, t)$ $\Longrightarrow$ bubble trap in the dressed picture (old m_F bad quantum number)

[Lundblad *et al.*, npj Microgravity **5**, 30 (2019)]

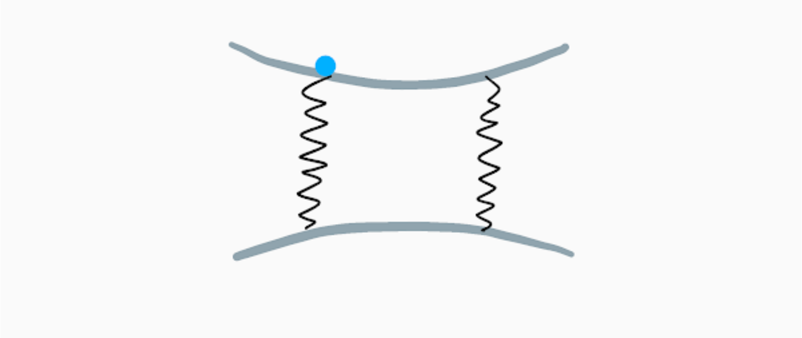
Radiofrequency-induced adiabatic potential



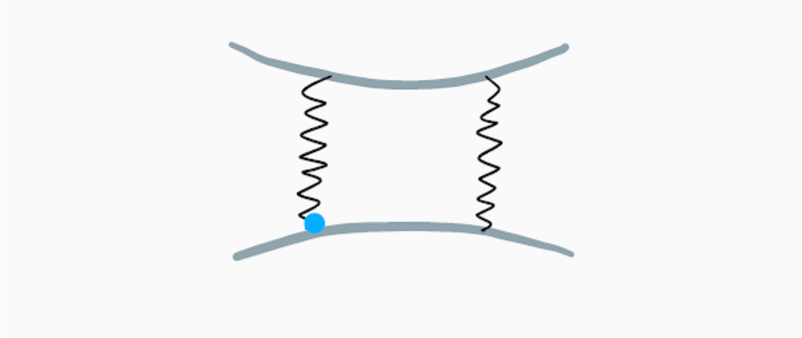
Radiofrequency-induced adiabatic potential



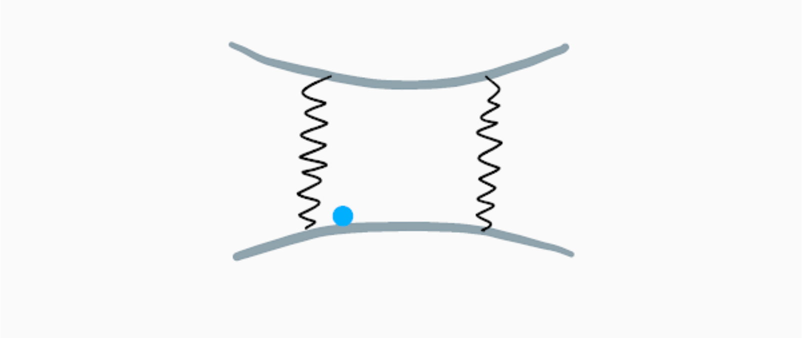
Radiofrequency-induced adiabatic potential



Radiofrequency-induced adiabatic potential



Radiofrequency-induced adiabatic potential



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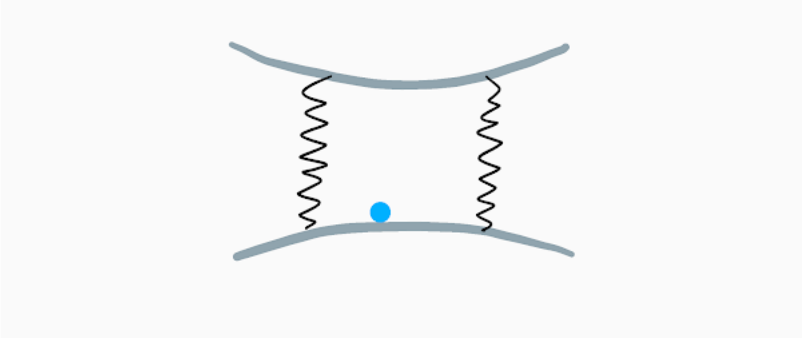
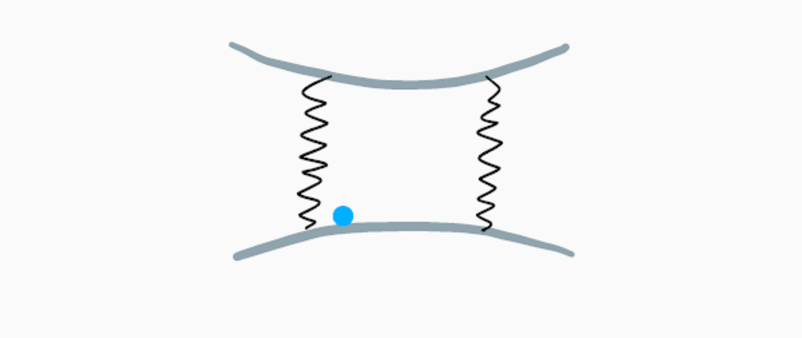
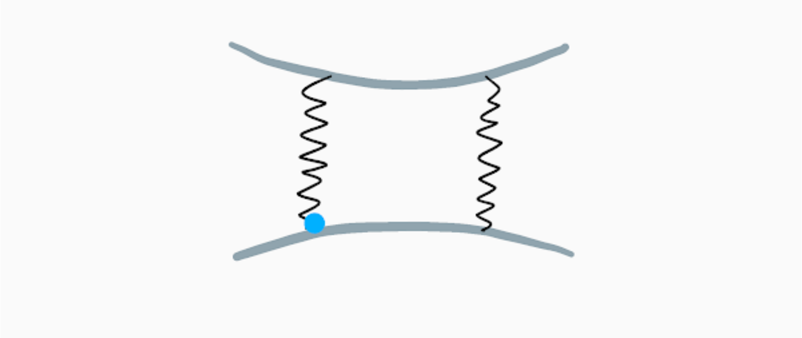


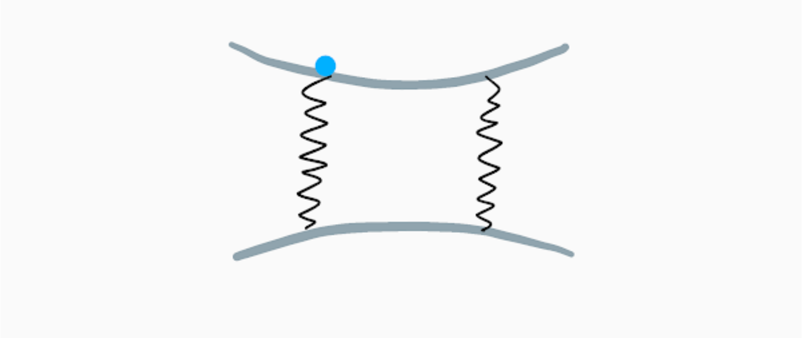
Radiofrequency-induced adiabatic potential



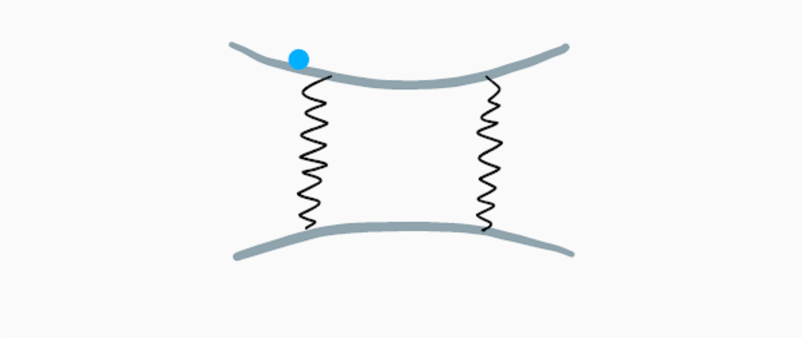
Radiofrequency-induced adiabatic potential



Radiofrequency-induced adiabatic potential



Radiofrequency-induced adiabatic potential



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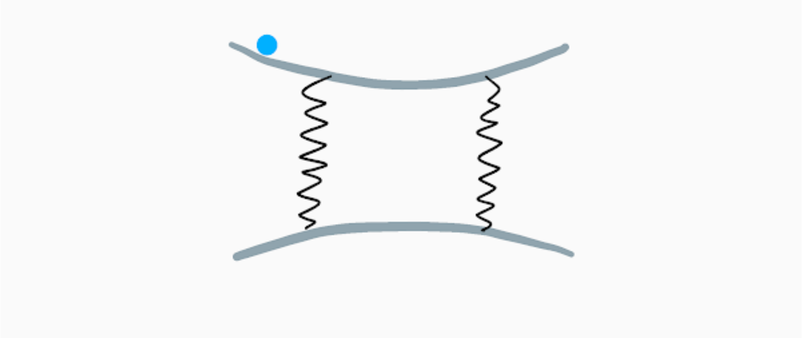


Radiofrequency-induced adiabatic potential

Bubble trap

$$U(\vec{r}) = M_F \sqrt{\left[\sum_i m\,\omega_i{}^2 x_i^2/2 - \hbar\Delta\right]^2 + (\hbar\Omega)^2},$$

ω_i: frequencies of the bare harmonic trap

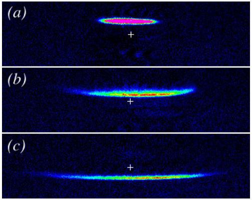
Δ: detuning from the resonant frequency

Ω: Rabi frequency between coupled levels

Minimum for $\omega_x{}^2 x^2 + \omega_y{}^2 y^2 + \omega_z{}^2 z^2 = 2\hbar\Delta/m$.

[Zobay, Garraway, Phys. Rev. Lett. **86**, 1195 (2001)]

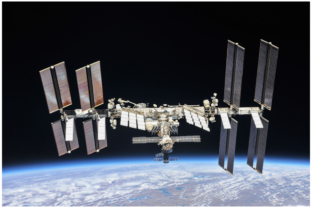
$$U(\vec{r}) \;=\; M_F \sqrt{\left[\sum_i m\,\omega_i{}^2 x_i^2/2 - \hbar\Delta\right]^2 + (\hbar\Omega)^2} \;+\; \underline{mgz}$$

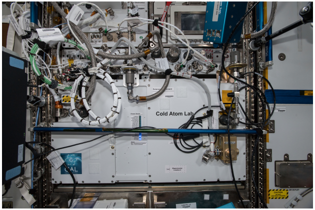


[Colombe *et al.*, EPL **67**, 593 (2004)]

If gravity is included the **atoms will fall to the bottom of the trap!**

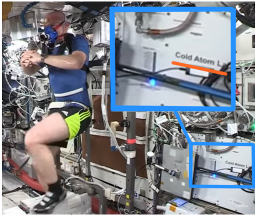
⇒ Experiments on NASA-JPL **Cold Atom Lab**, see [Elliott *et al.*, npj Microgravity **4**, 16 (2018)] (PI: N. Lundblad)





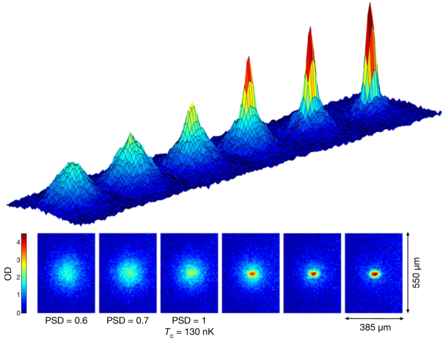
Cold Atom Lab (CAL)

This one ↓



2019 upgrade:

Routine production of microgravity BECs:



[Aveline *et al.*, Nature **582**, 193 (2020)]

...towards BECCAL:

[Frye *et al.*, EPJ Quantum Technol **8**, 1 (2021)]

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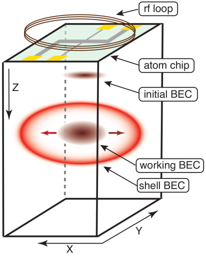
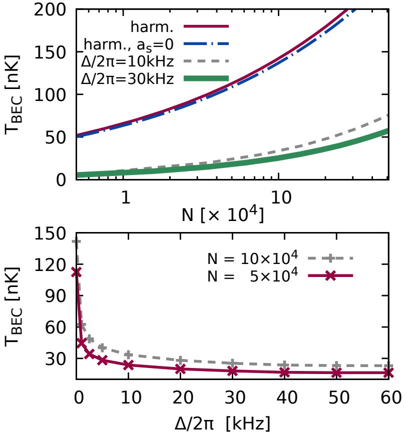
Properties and challenges of shell-shaped condensates

For the realistic trap parameters of NASA-JPL CAL experiment:

$$T_{BEC}^{bubble\ trap} \ll T_{BEC}^{harmonic\ trap}\ *$$



(*from Hartree-Fock theory [Giorgini *et al.* J. Low T. Phys. (1997)])



[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

Number density: $T = 0$ vs T_{BEC}

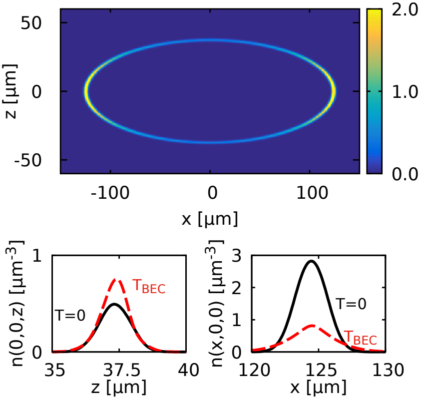
Gross-Pitaevskii equation at $T = 0$:

$$\left[-\frac{\hbar^2\nabla^2}{2m}+U(\vec{r})+g|\psi|^2\right]\psi = \mu\psi$$

Hartree-Fock at T_{BEC}:

$$n(\vec{r}) = \int \frac{d^3p}{e^{(E^{HF}(\vec{p},\vec{r}))/(k_B T_{\text{BEC}})} - 1}$$

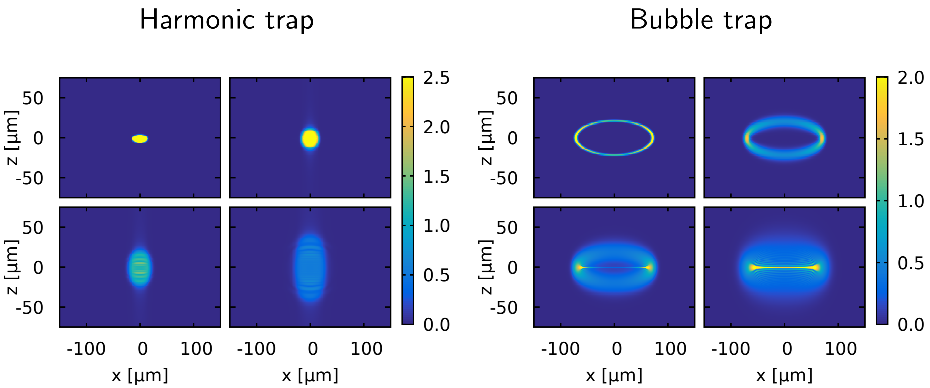
$$E^{HF}(\vec{p},\vec{r}) = p^2/(2m)+U(\vec{r})-\mu+2gn(\vec{r})$$



Number density as a probe of the system temperature

[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

Free expansion

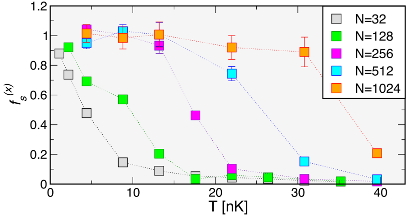


[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

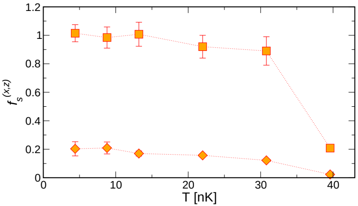
Path Integral Monte Carlo - superfluid fraction

Experimental trap parameters, but **strongly-interacting** bosons:

calculation of $f_s^{(i)} = I_i / I_{i,classical}$



$f_s^{(x)}$



$f_s^{(x)} > f_s^{(z)}$

(x: main symmetry axis; $y, z \perp x$)

[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

A take-home message

◇ Experiments can be challenging: to have a sufficient condensate fraction in $\sim 10^5$ atoms you need a final temperature $\ll 30\,\mathrm{nK}$.

$\Rightarrow$ It is worth studying the finite-temperature properties and BKT physics

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Bose-Einstein condensation on the surface of a sphere

Noninteracting case, single particle on a sphere of radius R:

$\frac{\hat{L}^2}{2mR^2}\,\psi_{l,m_l}(\theta,\varphi) = \epsilon_l\,\psi_{l,m_l}(\theta,\varphi)$,

with $\epsilon_l = \frac{\hbar^2}{2mR^2}\,l(l+1)$ and $m_l = -l,\ldots,+l$.

Particle number at temperature T:

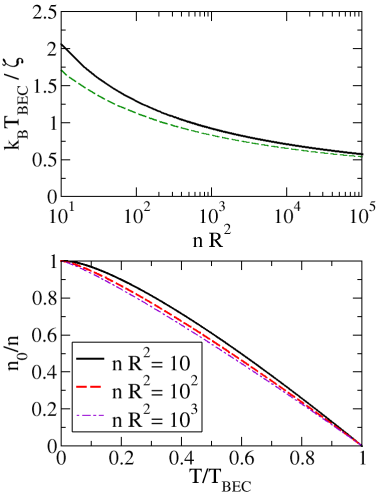
$$N = \sum_{l=0}^{+\infty}\sum_{m_l=-l}^{+l}\frac{1}{e^{(\epsilon_l-\mu)/(k_B T)}-1} = N_0 + \sum_{l=1}^{+\infty}\frac{2l+1}{e^{(\epsilon_l-\epsilon_0)/(k_B T)}-1}$$

when $N_0 = 0 \implies T = T_{\mathrm{BEC}}$

BEC on a sphere: **noninteracting case**

$$k_B T_{\text{BEC}} = \frac{\frac{2\pi\hbar^2}{m} n}{\frac{\beta_{\text{BEC}}\hbar^2}{mR^2} - \ln\left(e^{\beta_{\text{BEC}}\hbar^2/mR^2} - 1\right)}$$

$$\frac{n_0}{n} = 1 - \frac{1 - \frac{mR^2}{\hbar^2\beta}\ln\left(e^{\beta\hbar^2/mR^2} - 1\right)}{1 - \frac{mR^2}{\hbar^2\beta_{\text{BEC}}}\ln\left(e^{\beta_{\text{BEC}}\hbar^2/mR^2} - 1\right)}$$



[AT, Salasnich, PRL **123**, 160403 (2019)]

BEC on a sphere: **interacting case**

Popov theory to calculate the grand canonical potential:

$$\Omega = -\beta^{-1} \ln(\mathcal{Z}), \qquad \mathcal{Z} = \int \mathcal{D}[\bar{\psi}, \psi] \; e^{-S[\bar{\psi},\psi]/\hbar},$$

where

$$S[\bar{\psi}, \psi] = \int_0^{\beta\hbar} d\tau \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \, R^2 \sin(\theta) \, \mathcal{L}(\bar{\psi}, \psi)$$

is the Euclidean action, and

$$\mathcal{L} = \bar{\psi}(\theta, \varphi, \tau)\left(\hbar\partial_\tau + \frac{\hat{L}^2}{2mR^2} - \mu\right)\psi(\theta, \varphi, \tau) + \frac{g}{2}|\psi(\theta, \varphi, \tau)|^4$$

is the Euclidean Lagrangian.

(“dimensional” reduction in: [Móller *et al.*, NJP **22**, 063059 (2020)])

Thermodynamic potential Ω

In the Bose-condensed phase

$$\psi(\theta,\varphi,\tau)=\psi_0+\eta(\theta,\varphi,\tau)$$

Keeping up to $\sim\eta^2$ terms, expanding with spherical harmonics, and performing functional integration we get

$$\begin{aligned}\Omega(\mu,\psi_0^2) &= 4\pi R^2\big(-\mu\psi_0^2+g\psi_0^4/2\big)+\frac{1}{2}\sum_{l=1}^{\infty}\sum_{m_l=-l}^{l}E_l(\mu,\psi_0^2)\\ &+ \frac{1}{\beta}\sum_{l=1}^{\infty}\sum_{m_l=-l}^{l}\ln\Big(1-e^{-\beta E_l(\mu,\psi_0^2)}\Big),\end{aligned}$$

with $E_l(\mu,\psi_0^2)=\sqrt{(\epsilon_l-\mu+2g\psi_0^2)^2-g^2\psi_0^4}$.

Number density n

Following [Kleinert, Schmidt, Pelster PRL **93**, 160402 (2004)]

we impose $\frac{\partial\Omega}{\partial\psi_0}(\mu, \psi_0^2) = 0$, obtaining $\psi_0^2 = n_0(\mu)$

then, perturbatively $E_l^B(\mu, n_0(\mu)) = \sqrt{\epsilon_l(\epsilon_l + 2\mu)}$ and $\mu(n_0)$

Number density:

$$n(\mu) = -\frac{1}{4\pi R^2}\frac{\partial\Omega}{\partial\mu}(\mu, n_0(\mu)),$$

From $\mu(n_0)$ we calculate

$$n(\mu(n_0)) = n_0 + f_g^{(0)}(n_0) + f_g^{(T)}(n_0),$$

$f_g^{(0)}(n_0)$, $f_g^{(T)}(n_0)$: **analytical results!**

Critical temperature and condensate fraction

The critical temperature of the interacting system reads

$$k_B T_{\text{BEC}} = \frac{\frac{2\pi\hbar^2 n}{m} - \frac{gn}{2}}{\frac{\hbar^2\beta_{BEC}}{2mR^2}\left(1+\sqrt{1+\frac{2gmnR^2}{\hbar^2}}\right) - \ln\left(e^{\frac{\hbar^2\beta_{BEC}}{mR^2}\sqrt{1+\frac{2gmnR^2}{\hbar^2}}} - 1\right)}.$$

and the condensate fraction

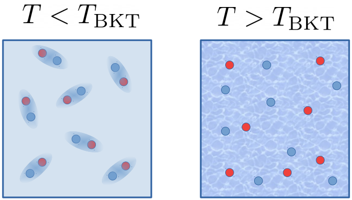
$$\frac{n_0}{n} = 1 - \frac{mg}{4\pi\hbar^2} - \frac{1}{4\pi R^2 n}\left[1+\sqrt{1+\frac{2gmnR^2}{\hbar^2}}\right] + \frac{mk_B T}{2\pi\hbar^2 n}\ln\left(e^{\frac{\hbar^2}{mR^2 k_B T}\sqrt{1+(2gmnR^2/\hbar^2)}} - 1\right).$$

$R \to \infty$: $T_{\text{BEC}} \to 0$, flat-case quantum depletion [Schick, PRA **3**, 1067 (1971)]

[AT, Salasnich, PRL **123**, 160403 (2019)]

BKT transition on a sphere

In the flat case: unbinding of vortex-antivortex dipoles at $T = T_{\text{BKT}}$ destroys the quasi long-range order.



[Ovrut, Thomas PRD **43**, 1314 (1991)]: Kosterlitz-Nelson criterion on a sphere

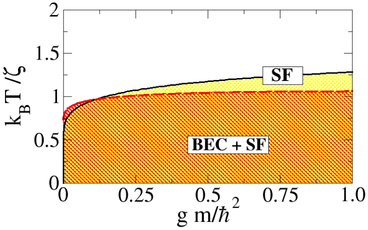
$$k_B T_{BKT} = \frac{\pi}{2} \frac{\hbar^2 n_s^{(0)}(T_{\text{BKT}})}{m},$$

with the bare superfluid density:

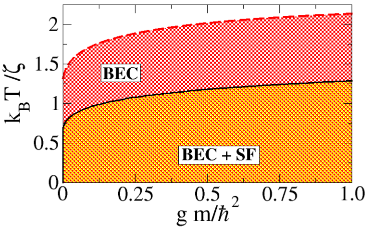
$$n_s^{(0)} = n - \frac{1}{k_B T} \int_1^{+\infty} dl \, \frac{(2l+1)}{4\pi R^2} \frac{\hbar^2 (l^2 + l)}{2mR^2} \frac{e^{E_l^B/(k_B T)}}{(e^{E_l^B/(k_B T)} - 1)^2}.$$

BEC and BKT on the sphere

Usual 2D picture (thermodyn. limit)
$nR^2 = 10^5$



Region of BEC only
$nR^2 = 10^2$



BEC transition (red dashed)
BKT=SF transition (**black**)

[AT, Salasnich, PRL **123**, 160403 (2019)]

BEC and BKT on the sphere

- is $nR^2 = 10^2$ observable? Yes!

- we used the Kosterlitz-Nelson criterion with $n_s^{(0)}$

$\Rightarrow$ current work!

Outline

- Bose-Einstein Condensation and Superfluidity
- Quantum statistical properties of shell-shaped BECs
  - Properties and challenges of shell-shaped condensates
  - Bose-Einstein condensation on the surface of a sphere
  - Recent developments
- Conclusions

Current work: **extending the RG equations for a spherical superfluid.**

Renormalization group equations in the flat case

Fundamental analogy:

Charges with 2D Coulomb interaction = Vortices in 2D a superfluid

- Screening of the interaction due to polarization
  $\rightarrow$ renormalization of the superfluid density: $n_s^{(0)}/\epsilon(r)$
- Define $y_0 = e^{-\beta\mu_{vortex}}$ and $K_0 = \frac{\hbar^2 n_s^{(0)}}{m k_B T}$
- Running of the parameters in $l = \ln(r/a)$
  $\rightarrow$ y(l) and $K(l) = \frac{\hbar^2}{m k_B T}\frac{n_s^{(0)}}{\epsilon(r)}$

Current work: **extending the RG equations for a spherical superfluid.**

Renormalization group equations in the flat case

– Perturbative calculation in y_0 of $\epsilon(r)$ leads to RG equations:

$$\begin{aligned}\frac{dK^{-1}(l)}{dl} &= -4\pi^3 y^2(l) + o(y_0^3)\\ \frac{dy(l)}{dl} &= [2-\pi K(l)]\, y(l) + o(y_0^2)\end{aligned}$$

If T is such that $2-\pi K < 0$

$\Rightarrow \frac{dy(l)}{dl} < 0$ and $y(\infty) = 0$ $\Rightarrow$ no free vortices

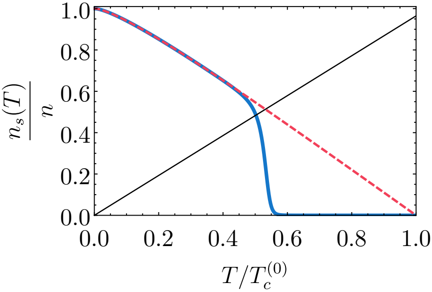
Proliferation of vortices at: $K_{\mathrm{BKT}} = 2/\pi \leftrightarrow \frac{n_s(T^-_{\mathrm{BKT}})}{T_{\mathrm{BKT}}} = \frac{2}{\pi}\frac{mk_B}{\hbar^2}$

[Kosterlitz, Thouless, J. Phys. C **6**, 1181 (1973)] [Kosterlitz, J. Phys. C **7**, 1046 (1974)] [Nelson, Kosterlitz, PRL **39**, 1201 (1977)]

Renormalized superfluid density of a spherical superfluid

Extending the Kosterlitz-Thouless RG equations on a spherical superfluid...

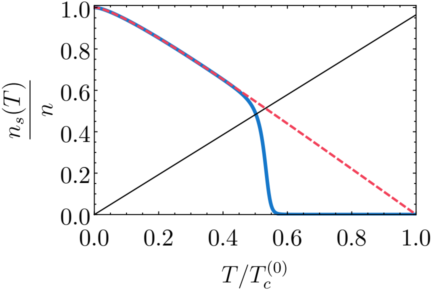
Renormalized superfluid density (**preliminary**)



[Tononi, Pelster, Salasnich, *in preparation*]

Renormalized superfluid density of a spherical superfluid

Different from the infinite flat case, **the transition is not sharp** and there are finite-size nonuniversal corrections.



[Tononi, Pelster, Salasnich, *in preparation*]

Current goals:

◇ Investigation of BEC-BKT interplay, finite-size corrections, ...

Outline

- ▷ Bose-Einstein Condensation and Superfluidity
- ▷ Quantum statistical properties of shell-shaped BECs
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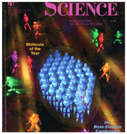
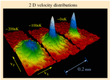
Conclusions

Bose-Einstein Condensation

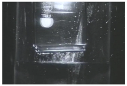


Bose-Einstein condensate:
a many-body system of identical bosonic particles of which a **macroscopic fraction** occupies the same lowest-energy **single-particle state**

Cornell & Wieman, Ketterle, in 1995:
observation of **Bose-Einstein condensation** of ^{87}Rb and ^{23}Na gases through laser cooling and evaporative cooling



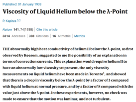
Superfluidity



Superfluidity:
frictionless flow of a quantum liquid through narrow capillaries

Kapitza, in 1938:
observation of **superfluidity** in liquid ^{4}He below $T_\lambda = 2.17$ K

Landau & Tisza, in 1941:
two-fluid model


Published: 01 January 1938

Viscosity of Liquid Helium below the λ-Point

P. Kapitza

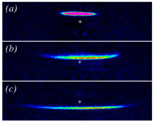
Nature **141**, 74(1938) | Cite this article

3314 Accesses | **388** Citations | **16** Altmetric | Metrics

THE abnormally high heat conductivity of helium II below the λ-point, as first observed by Keesom, suggested to me the possibility of an explanation in terms of convection currents. This explanation would require helium II to have an abnormally low viscosity; at present, the only viscosity measurements on liquid helium have been made in Toronto[1], and showed that there is a drop in viscosity below the λ-point by a factor of 3 compared with liquid helium at normal pressure, and by a factor of 8 compared with the value just above the λ-point. In these experiments, however, no check was made to ensure that the motion was laminar, and not turbulent.

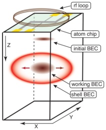
The experiments with "bubble" traps

technically difficult on Earth...



[Colombe *et al.*, EPL **67**, 593 (2004)]

NASA-JPL Cold Atom Laboratory



[Lundblad *et al.*, npj Microgravity **5**, 30 (2019)]

Bose-Einstein condensation on the surface of a sphere

Noninteracting case, single particle on a sphere of radius R:

$$\frac{\hat{L}^2}{2mR^2}\,\psi_{l,m_l}(\theta,\varphi) = \epsilon_l\,\psi_{l,m_l}(\theta,\varphi),$$

with $\epsilon_l = \frac{\hbar^2}{2mR^2}\,l(l+1)$ and $m_l = -l, \ldots, +l$.

Particle number at temperature T:

$$N = \sum_{l=0}^{+\infty}\sum_{m_l=-l}^{+l} \frac{1}{e^{(\epsilon_l-\mu)/(k_B T)}-1} = N_0 + \sum_{l=1}^{+\infty} \frac{2l+1}{e^{(\epsilon_l-\epsilon_0)/(k_B T)}-1}$$

when $N_0 = 0 \implies T = T_{\mathrm{BEC}}$

Conclusions: highlights

- ◇ Phase of BEC without superfluidity!
- ◇ Finite-size **nonuniversal corrections to BKT** physics.
- ◇ For the future: vortices, dynamical properties...

Thank you for your attention!

References

A. Tononi, F. Cinti, and L. Salasnich, *Quantum Bubbles in Microgravity*, Physical Review Letters **125**, 010402 (2020).

A. Tononi and L. Salasnich, *Bose-Einstein Condensation on the Surface of a Sphere*, Physical Review Letters **123**, 160403 (2019).

A. Tononi, A. Pelster, and L. Salasnich, in preparation.