Binding of heavy fermions by a single light atom in 1D

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based on

[A. Tononi, J. Givois, and D. S. Petrov, arXiv:2205.01018]

Outline

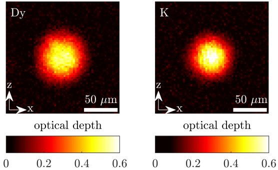
- ▷ Introduction and motivation
- ▷ Our results: bound states of $N+1$ fermions in 1D
- ▷ Comments and derivation of the results
- ▷ Conclusions and perspectives

Introduction and motivation

Experiments* with mixtures of fermionic atoms with large mass imbalance:

$^{173}\mathrm{Yb}$–$^{6}\mathrm{Li}$, $^{53}\mathrm{Cr}$–$^{6}\mathrm{Li}$, $^{40}\mathrm{K}$–$^{6}\mathrm{Li}$, $^{161}\mathrm{Dy}$–$^{40}\mathrm{K}$

$^{161}\mathrm{Dy}$–$^{40}\mathrm{K}$ mixture:



[Ravensbergen et al, PRA **98**, 063624 (2018)]

*: [Green et al, PRX **10**, 031037 (2020)] [Neri et al, PRA **101**, 063602 (2020)] [Wille et al, PRL **100**, 053201 (2008)] [Taglieber et al, PRL. **100**, 010401 (2008)] [Voigt et al, PRL **102**, 020405 (2009)] [Green et al, PRX **10**, 031037 (2020)] [Ravensbergen et al, PRL **124**, 203402 (2020)]

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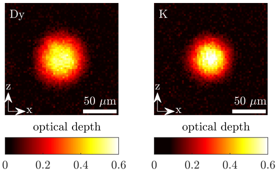

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- it is possible to cool and confine the atoms with optical and magnetic potentials, producing one-dimensional configurations

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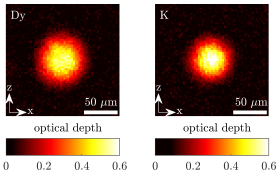

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Experiments* with mixtures of fermionic atoms with large mass imbalance:

^{173}Yb–^{6}Li, ^{53}Cr–^{6}Li, ^{40}K–^{6}Li, ^{161}Dy–^{40}K

- it is possible to cool and confine the atoms with optical and magnetic potentials, producing one-dimensional configurations
- It is also possible to tune the interatomic interactions with magnetic fields

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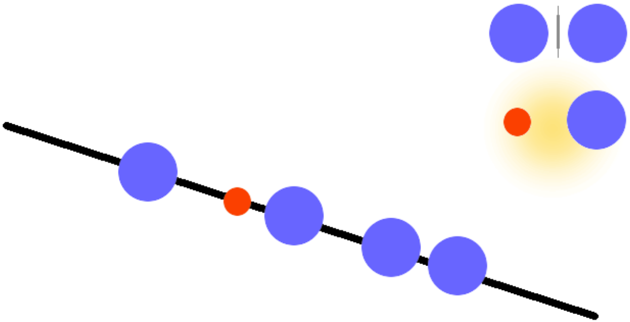
Our system

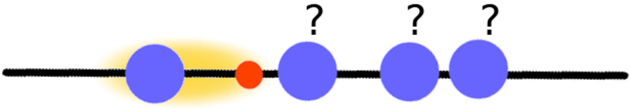
N heavy fermions of mass M and 1 light atom of mass m.



In this system, there is a competition between:

- Heavy-heavy repulsion due to Pauli exclusion principle
- Heavy-light attraction

How many heavy atoms can be bound by a single light atom in 1D?



Relevant scales of the 1D problem:

- mass ratio M/m,
- number of heavy atoms N,
- scattering length a (determining the 1+1 binding energy)

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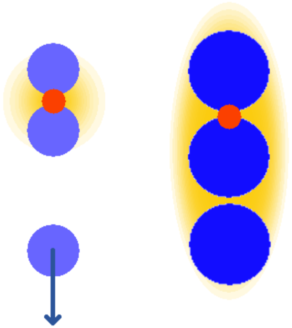
Bound states of $N+1$ fermions in 1D

Increasing the mass ratio M/m:
the attraction due to the exchange of
the light atom wins over Pauli pressure

$\Rightarrow$ More and more atoms can be bound
into a $N+1$ cluster.



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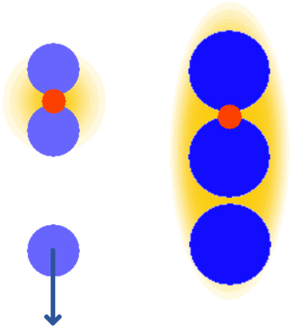

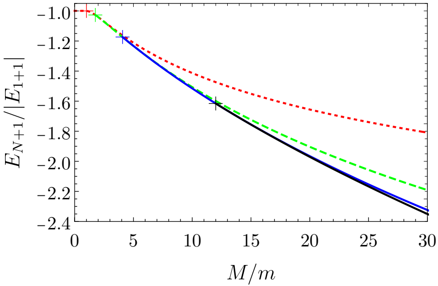

This is indeed shown by the **exact solution of the quantum mechanical problem** up to $N=5$.

($N = 2, 3, 4, 5$ here)

[A. Tononi, J. Givois, and D. S. Petrov, arXiv:2205.01018]

Bound states of $N+1$ fermions in 1D

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Bound states of $N + 1$ fermions in 1D

What about the large N limit?

We develop:

- a mean-field theory based on the **Thomas-Fermi** approximation for the heavy fermions
- a **Hartree-Fock** theory

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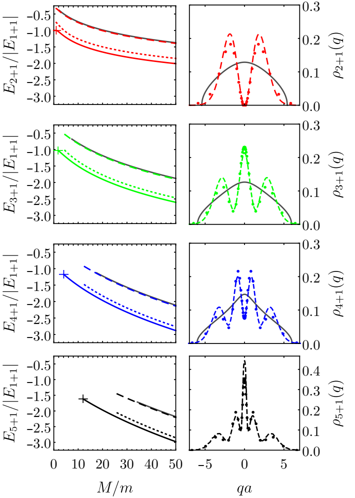
Bound states of $N+1$ fermions in 1D

The Hartree Fock theory works also for small N:



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Quantitative agreement with exact energies and correlations.

Let us see more in detail...

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Exact results: the STM equation

Schrödinger equation for a system of N heavy plus 1 light fermions:

$$\left[-\sum_{i=1}^{N}\frac{\partial^2_{x_i}}{2M}-\frac{\partial^2_{x_{N+1}}}{2m}+g\sum_{i<N+1}\delta(x_i-x_{N+1})-E\right]\psi(x_1,...,x_N,x_{N+1})=0,$$

where $E<0$, and $g<0$.

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where $E<0$, and $g<0$.

We consider $\psi(x_1,...,x_{N-1},x_N,x_{N+1}=x_N)$, and its Fourier transform $F(q_1,...,q_{N-1})$ satisfies the STM equation

$$\left[\frac{a}{2}-\frac{1}{2\kappa(q_1,...,q_{N-1})}\right]F(q_1,...,q_{N-1})=-\int\frac{dp}{2\pi}\frac{\sum_{j=1}^{N-1}F(q_1,...,q_{j-1},p,q_{j+1},...,q_{N-1})}{\kappa^2(q_1,...,q_{N-1})+(p+\frac{m_r}{m}\sum_{i=1}^{N-1}q_i)^2},$$

where $\kappa(q_1,...q_{N-1})=\sqrt{-2m_rE+\frac{m_r}{M+m}(\sum_{i=1}^{N-1}q_i)^2+\frac{m_r}{M}\sum_{i=1}^{N-1}q_i^2}$.

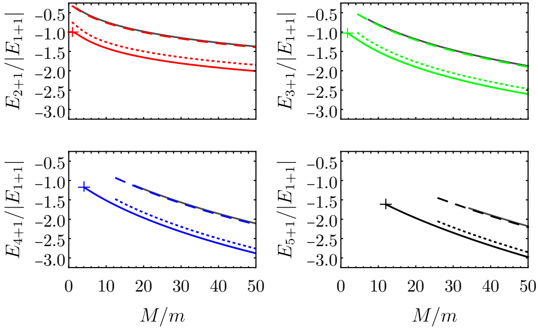
Integro-differential equation that can be solved as a matrix one.

[Skorniakov, Ter-Martirosian, JETP **4**, 648 (1957)]

[Pricoupenko, Petrov, PRA **100**, 042707 (2019)]

Exact results: obtained through STM equation

The exact solution of the STM equation gives the energies (continuous lines) of the $N+1$ clusters:



With a detailed analysis, we identify the critical mass ratios:

$(M/m)_{2+1} = 1$, $(M/m)_{3+1} = 1.76$,
$(M/m)_{4+1} = 4.2$, $(M/m)_{5+1} = 12.0$

Two questions

- Large N limit?
- Are there computationally-cheap methods that work also at small N?

Thomas-Fermi approach

In the large N limit: mean-field theory based on the Thomas-Fermi approximation for the heavy fermions

$$\Omega = \int \left[\frac{|\phi'(x)|^2}{2m} + gn(x)|\phi(x)|^2 + \frac{\pi^2 n^3(x)}{6M} - \epsilon|\phi(x)|^2 - \mu n(x) \right] dx,$$

and $\phi(x)$ and $n(x)$ are obtained minimizing Ω.

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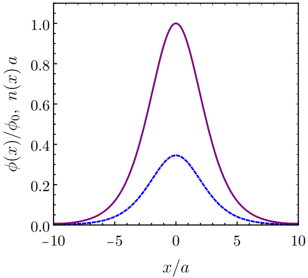
and $\phi(x)$ and $n(x)$ are obtained minimizing Ω.

When $\mu = 0$ (threshold for binding a new heavy atom), we find analytical solutions:

$$\phi(x) = \frac{-3\pi\epsilon}{\sqrt{-8Mg^3}} \frac{1}{\cosh^2(\sqrt{-m\epsilon/2}x)}$$

$$n(x) = \sqrt{-2Mg/\pi^2}\,|\phi(x)|$$

Threshold: $\left(\frac{M}{m}\right)_{N+1} = \frac{\pi^2}{36} N^3$



Thomas-Fermi approach

We then extend the theory for $\mu \neq 0$, and calculate the energies of the clusters.

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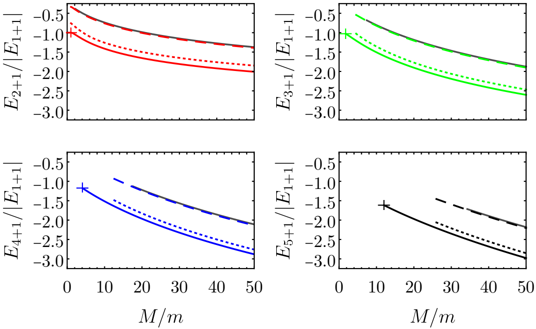
Thomas-Fermi approach (grey curves):



The Thomas Fermi approach is: analytical, computationally cheap, good at large N, **but** it does not match the small-N exact results.

Hartree-Fock approach

We obtain and solve the following equations:

$$-\partial_x^2 \phi_1(x)/2m + gn(x)\phi_1(x) = \epsilon_1 \phi_1(x),$$
$$-\partial_x^2 \Psi_\nu(x)/2M + g|\phi_1(x)|^2 \Psi_\nu(x) = E_\nu \Psi_\nu(x),$$

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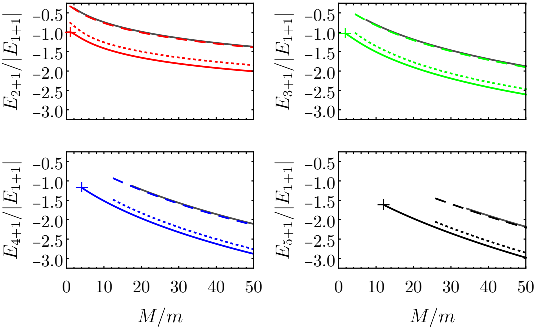

→ no improvement wrt TF energies (dashed lines), but a second-order correction (dotted lines) gives very good agreement

$N - 1$ atoms momentum distribution

All methods give access to the following quantity:

$$\rho_{N+1}(q) = \int |F(q, q_2, ..., q_{N-1})|^2 \, dq_2 ... dq_{N-1},$$

that can be used to compare their effectiveness at small N.

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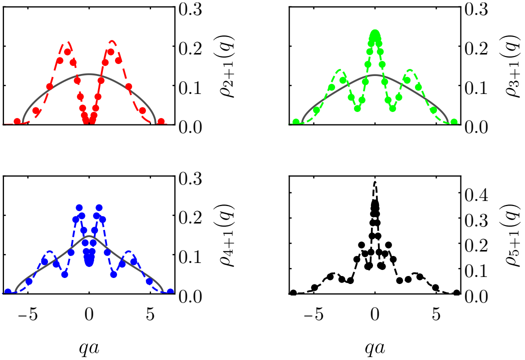
We find that Hartree-Fock reproduces very well these momentum correlations:



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- Exact results up to $N = 5$
- **TF theory**: analytical and works for large N
- **HF theory**: reproduces well energy and correlations at small and large N

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Possible generalization to other setups:

- higher dimensions
- more particles
- more components

Thank you for your attention!

References:

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— A. Pricoupenko, and D. S. Petrov, PRA **100**, 042707 (2019)