Quantum statistical properties of shell-shaped Bose-Einstein condensates

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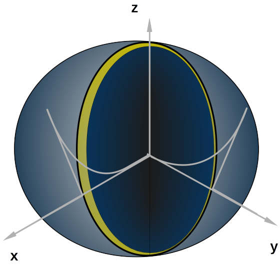
Shell-shaped Bose-Einstein condensate



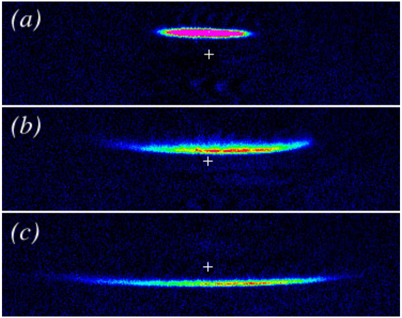
for strong radial confinement:
BEC on the surface of an ellipsoid

Why should we care?

- BEC in 2D (finite size)
- curved quantum system
- BKT, topology, vortices
- experimentally realizable

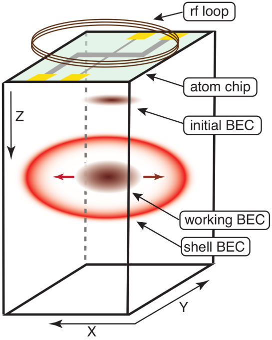
The experiments with “bubble” traps

technically difficult on Earth...



[Colombe *et al.*, EPL **67**, 593 (2004)]

NASA-JPL Cold Atom Laboratory

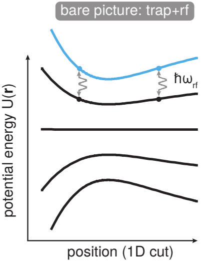


[Lundblad *et al.*, npj Microgravity **5**, 30 (2019)]

Outline

- Introduction on bubble traps
- Bose-Einstein condensation on the surface of a sphere
- Properties and challenges of shell-shaped condensates
- Summary and outlook

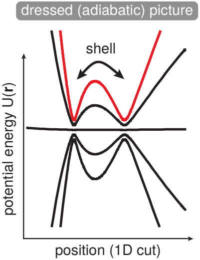
Introduction on bubble traps



Alkali-metal atoms
(here: total angular momentum $F = 2$)

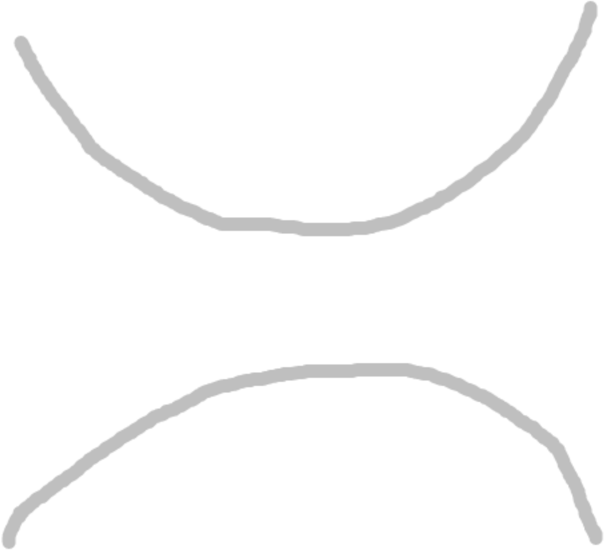
+ Magnetic field $\mathbf{B}(\vec{r})$
$\Longrightarrow$ space-dependent Zeeman splitting with $m_F = \{\pm 2, \pm 1, 0\}$ $\Longrightarrow$ space-dependent bare potentials $u(\vec{r})$

+ Radiofrequency field $\mathbf{B}_{\mathrm{rf}}(\vec{r}, t)$ $\Longrightarrow$ bubble trap in the dressed picture (old m_F bad quantum number)

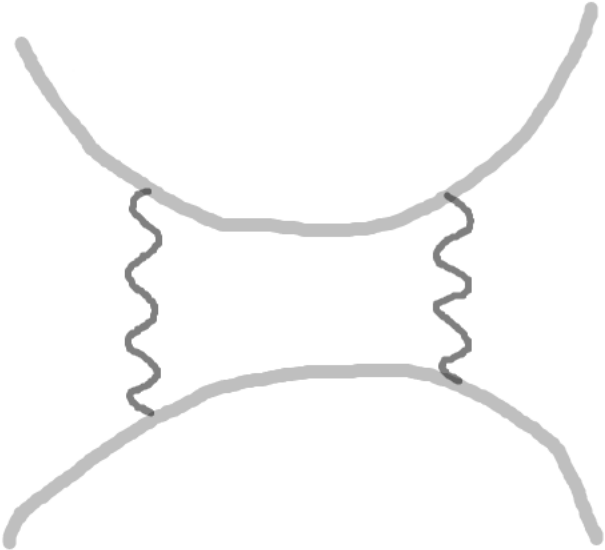


[Lundblad *et al.*, npj Micrograving **5**, 30 (2019)]

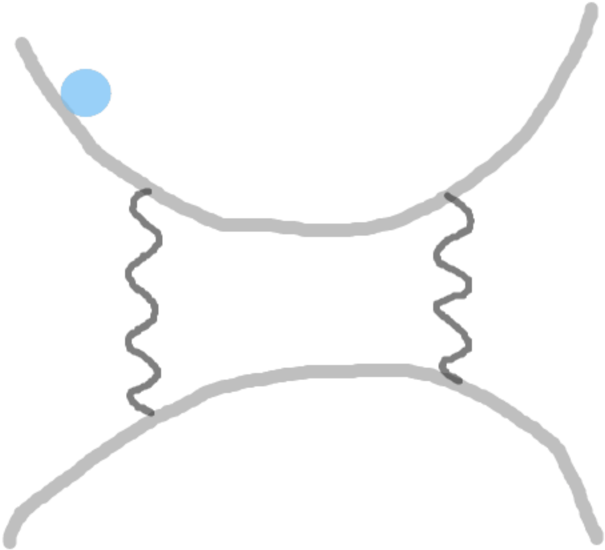
Radiofrequency-induced adiabatic potential



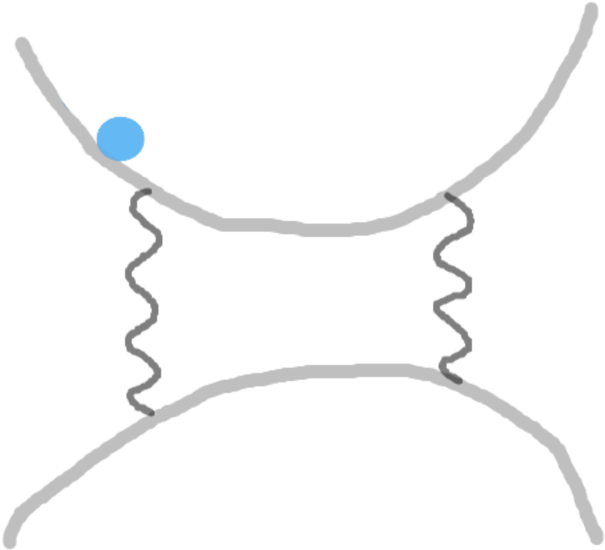
Radiofrequency-induced adiabatic potential



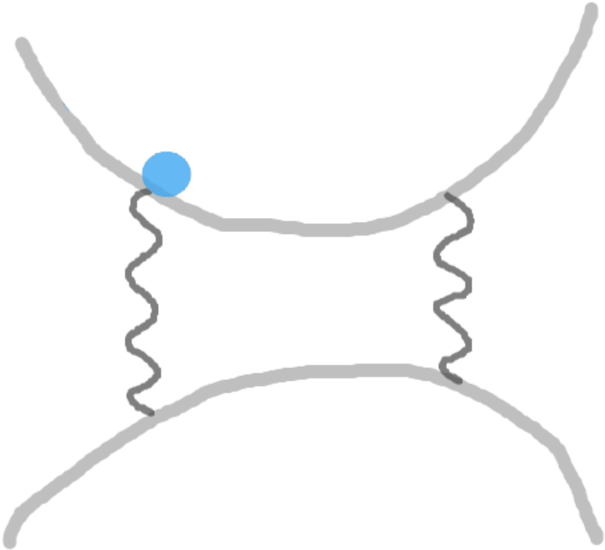
Radiofrequency-induced adiabatic potential



Radiofrequency-induced adiabatic potential



Radiofrequency-induced adiabatic potential



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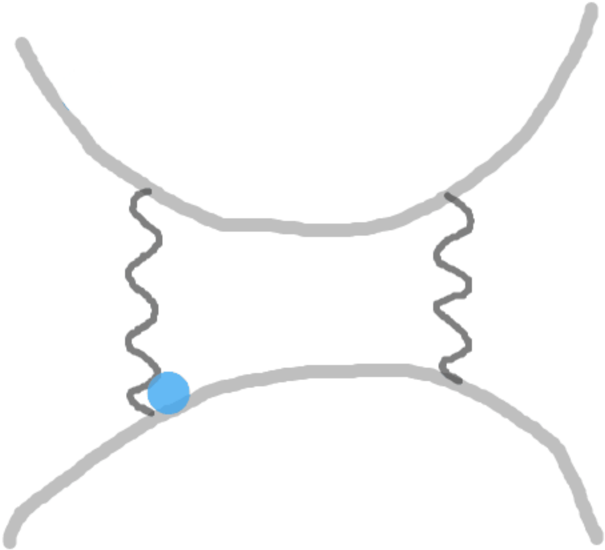
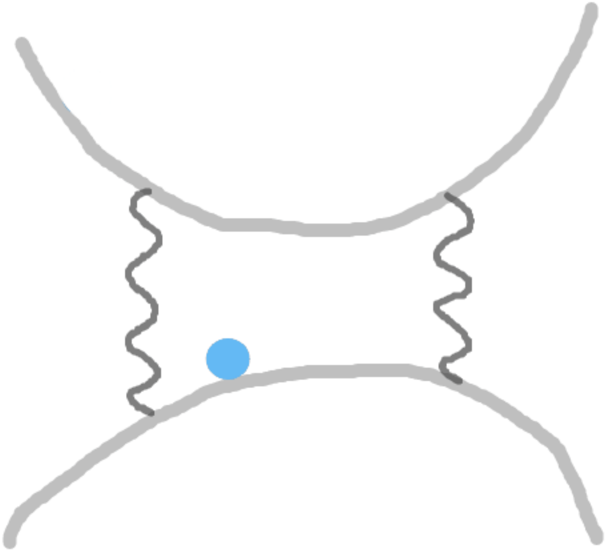


Radiofrequency-induced adiabatic potential



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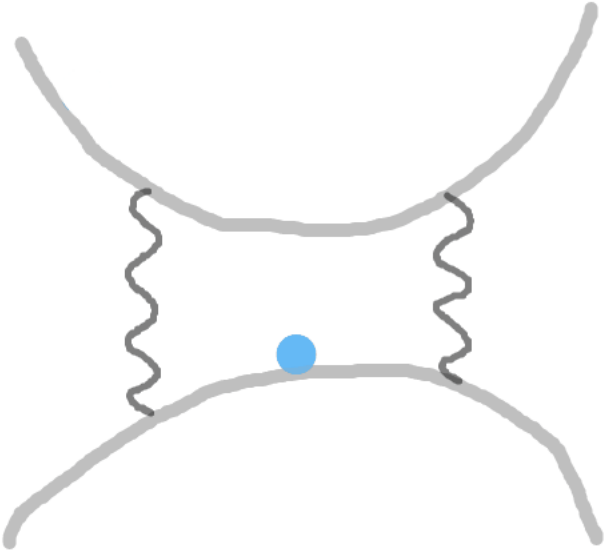
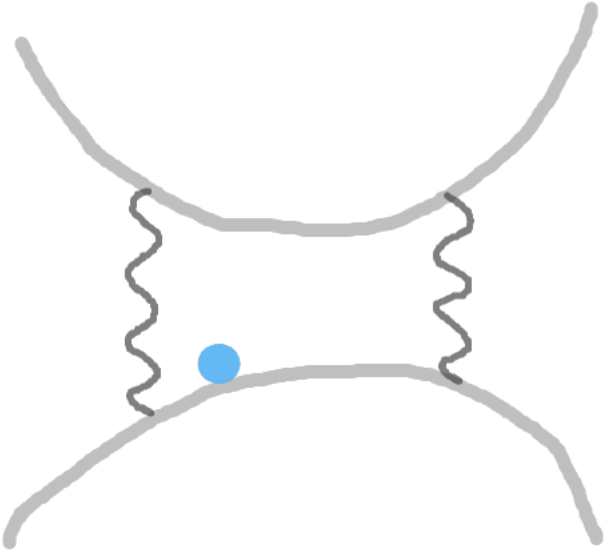


Radiofrequency-induced adiabatic potential



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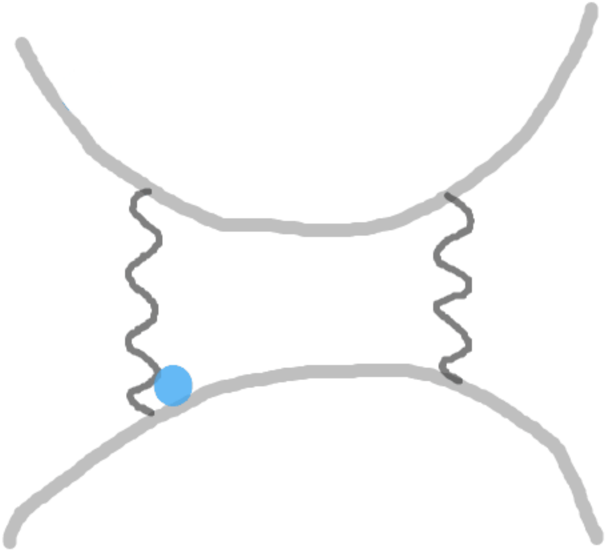

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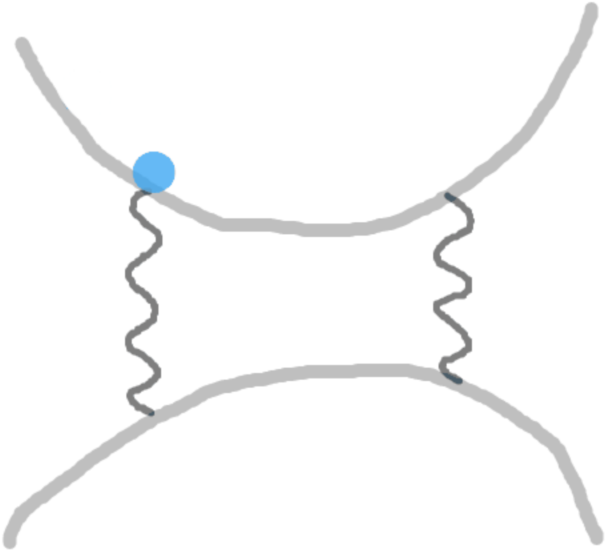

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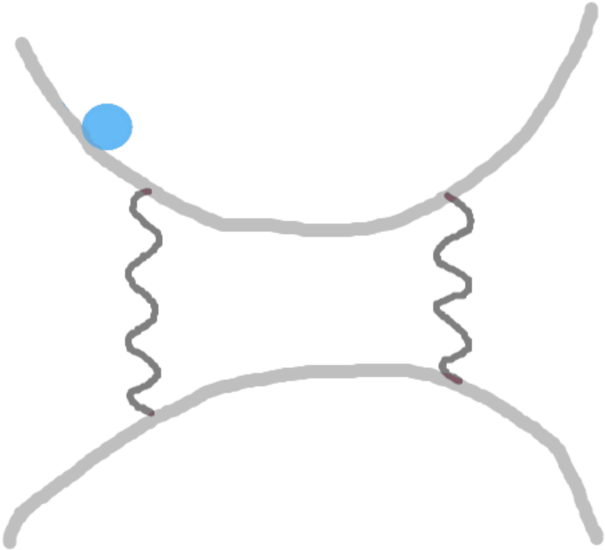


Radiofrequency-induced adiabatic potential

Bubble trap

$$U(\vec{r}) = M_F \sqrt{\left[\sum_i m\,\omega_i{}^2 x_i^2/2 - \hbar\Delta\right]^2 + (\hbar\Omega)^2},$$

ω_i: frequencies of the bare harmonic trap

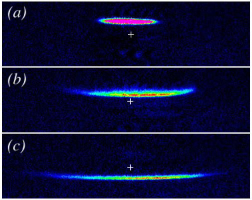
Δ: detuning from the resonant frequency

Ω: Rabi frequency between coupled levels

Minimum for $\omega_x{}^2 x^2 + \omega_y{}^2 y^2 + \omega_z{}^2 z^2 = 2\hbar\Delta/m$.

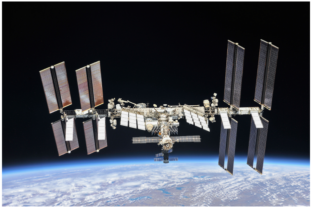
[Zobay, Garraway, Phys. Rev. Lett. **86**, 1195 (2001)]

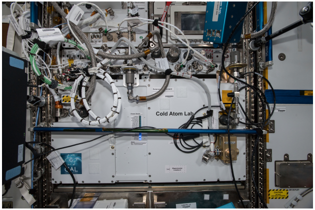
$$U(\vec{r}) = M_F \sqrt{\left[\sum_i m{\omega_i}^2 x_i^2/2 - \hbar\Delta\right]^2 + (\hbar\Omega)^2} + \underline{mgz}$$



If gravity is included the **atoms will fall to the bottom of the trap!**

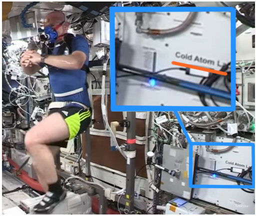
⇒ Experiments on NASA-JPL **Cold Atom Lab**, see [Elliott *et al.*, npj Microgravity **4**, 16 (2018)] (PI: N. Lundblad)





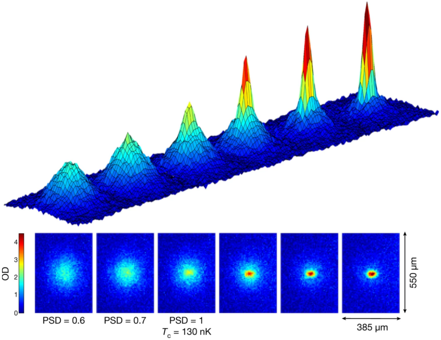
Cold Atom Lab (CAL)

This one ↓



2019 upgrade:

Routine production of microgravity BECs:



[Aveline *et al.*, Nature **582**, 193 (2020)]

...towards BECCAL:

[Frye *et al.*, arXiv:1912.04849]

Outline

- ▷ Introduction on bubble traps
- ▷ Bose-Einstein condensation on the surface of a sphere
- ▷ Properties and challenges of shell-shaped condensates
- ▷ Summary and outlook

Bose-Einstein condensation on the surface of a sphere

Noninteracting case, single particle on a sphere of radius R:

$\frac{\hat{L}^2}{2mR^2}\,\psi_{l,m_l}(\theta,\varphi) = \epsilon_l\,\psi_{l,m_l}(\theta,\varphi)$,

with $\epsilon_l = \frac{\hbar^2}{2mR^2}\,l(l+1)$ and $m_l = -l,\ldots,+l$.

Particle number at temperature T:

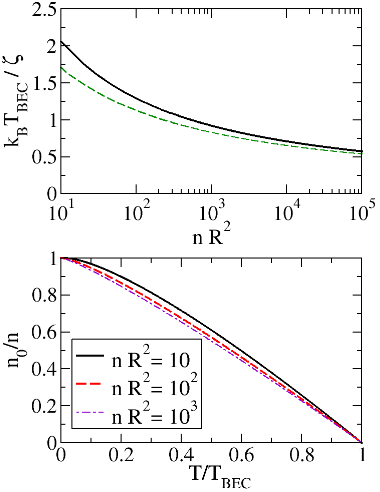
$$N = \sum_{l=0}^{+\infty}\sum_{m_l=-l}^{+l}\frac{1}{e^{(\epsilon_l-\mu)/(k_B T)}-1} = N_0 + \sum_{l=1}^{+\infty}\frac{2l+1}{e^{(\epsilon_l-\epsilon_0)/(k_B T)}-1}$$

when $N_0 = 0 \implies T = T_{\mathrm{BEC}}$

BEC on a sphere: **noninteracting case**

$$k_B T_{\text{BEC}} = \frac{\frac{2\pi\hbar^2}{m} n}{\frac{\beta_{\text{BEC}}\hbar^2}{mR^2} - \ln\left(e^{\beta_{\text{BEC}}\hbar^2/mR^2} - 1\right)}$$

$$\frac{n_0}{n} = 1 - \frac{1 - \frac{mR^2}{\hbar^2\beta} \ln\left(e^{\beta\hbar^2/mR^2} - 1\right)}{1 - \frac{mR^2}{\hbar^2\beta_{\text{BEC}}} \ln\left(e^{\beta_{\text{BEC}}\hbar^2/mR^2} - 1\right)}$$



[AT, Salasnich, PRL **123**, 160403 (2019)]

BEC on a sphere: **interacting case**

Popov theory to calculate the grand canonical potential:

$$\Omega = -\beta^{-1} \ln(\mathcal{Z}), \qquad \mathcal{Z} = \int \mathcal{D}[\bar{\psi}, \psi]\, e^{-S[\bar{\psi},\psi]/\hbar},$$

where

$$S[\bar{\psi}, \psi] = \int_0^{\beta\hbar} d\tau \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta\, R^2 \sin(\theta)\, \mathcal{L}(\bar{\psi}, \psi)$$

is the Euclidean action, and

$$\mathcal{L} = \bar{\psi}(\theta, \varphi, \tau)\left(\hbar\partial_\tau + \frac{\hat{L}^2}{2mR^2} - \mu\right)\psi(\theta, \varphi, \tau) + \frac{g}{2}|\psi(\theta, \varphi, \tau)|^4$$

is the Euclidean Lagrangian.

("dimensional" reduction in: [Móller *et al.*, NJP **22**, 063059 (2020)])

Thermodynamic potential Ω

In the Bose-condensed phase

$$\psi(\theta,\varphi,\tau) = \psi_0 + \eta(\theta,\varphi,\tau)$$

Keeping up to $\sim \eta^2$ terms, expanding with spherical harmonics, and performing functional integration we get

$$\begin{aligned}
\Omega(\mu,\psi_0^2) &= 4\pi R^2\big(-\mu\psi_0^2 + g\psi_0^4/2\big) + \frac{1}{2}\sum_{l=1}^{\infty}\sum_{m_l=-l}^{l} E_l(\mu,\psi_0^2) \\
&+ \frac{1}{\beta}\sum_{l=1}^{\infty}\sum_{m_l=-l}^{l} \ln\Big(1 - e^{-\beta E_l(\mu,\psi_0^2)}\Big),
\end{aligned}$$

with $E_l(\mu,\psi_0^2) = \sqrt{(\epsilon_l - \mu + 2g\psi_0^2)^2 - g^2\psi_0^4}$.

Number density n

Following [Kleinert, Schmidt, Pelster PRL **93**, 160402 (2004)]

we impose $\frac{\partial\Omega}{\partial\psi_0}(\mu,\psi_0^2)=0$, obtaining $\psi_0^2=n_0(\mu)$

then, perturbatively $E_l(\mu,n_0(\mu))=\sqrt{\epsilon_l(\epsilon_l+2\mu)}$ and $\mu(n_0)$

Number density:

$$n(\mu)=-\frac{1}{4\pi R^2}\frac{\partial\Omega}{\partial\mu}(\mu,n_0(\mu)),$$

From $\mu(n_0)$ we calculate

$$n(\mu(n_0))=n_0+f_g^{(0)}(n_0)+f_g^{(T)}(n_0),$$

$f_g^{(0)}(n_0)$, $f_g^{(T)}(n_0)$: **analytical results!**

Critical temperature and condensate fraction

The critical temperature of the interacting system reads

$$k_B T_{\text{BEC}} = \frac{\frac{2\pi\hbar^2 n}{m} - \frac{gn}{2}}{\frac{\hbar^2 \beta_{BEC}}{2mR^2}\left(1 + \sqrt{1 + \frac{2gmnR^2}{\hbar^2}}\right) - \ln\left(e^{\frac{\hbar^2\beta_{BEC}}{mR^2}\sqrt{1+\frac{2gmnR^2}{\hbar^2}}} - 1\right)}.$$

and the condensate fraction

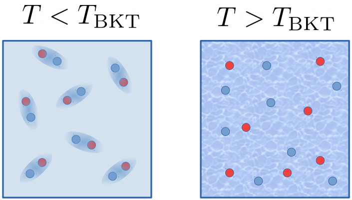
$$\frac{n_0}{n} = 1 - \frac{mg}{4\pi\hbar^2} - \frac{1}{4\pi R^2 n}\left[1 + \sqrt{1 + \frac{2gmnR^2}{\hbar^2}}\right] + \frac{mk_B T}{2\pi\hbar^2 n}\ln\left(e^{\frac{\hbar^2}{mR^2 k_B T}\sqrt{1+(2gmnR^2/\hbar^2)}} - 1\right).$$

$R \to \infty$: $T_{\text{BEC}} \to 0$, Schick result for quantum depletion.

[AT, Salasnich, PRL **123**, 160403 (2019)]

BKT transition on a sphere

The unbinding of vortex-antivortex dipoles at $T = T_{\text{BKT}}$ destroys the quasi long-range order.



[Ovrut, Thomas PRD **43**, 1314 (1991)]: Kosterlitz-Nelson criterion on a sphere

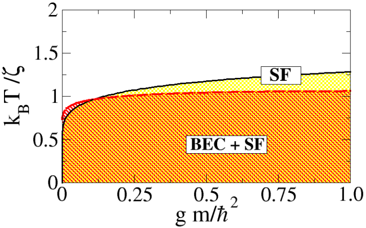
$$k_B T_{BKT} = \frac{\pi}{2} \frac{\hbar^2 n_s^{(0)}(T_{\text{BKT}})}{m},$$

with the bare superfluid density:

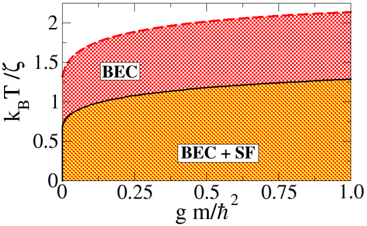
$$n_s^{(0)} = n - \frac{1}{k_B T} \int_1^{+\infty} \frac{dl\,(2l+1)}{4\pi R^2} \frac{\hbar^2(l^2+l)}{2mR^2} \frac{e^{E_l^B/(k_B T)}}{(e^{E_l^B/(k_B T)} - 1)^2}.$$

BEC and BKT on the sphere

Usual 2D picture (thermodyn. limit)
$nR^2 = 10^5$



Region of BEC only
$nR^2 = 10^2$



BEC transition (red dashed)
BKT=SF transition (**black**)

[AT, Salasnich, PRL **123**, 160403 (2019)]

BEC and BKT on the sphere

- we used the Kosterlitz-Nelson criterion with $n_s^{(0)}$
- is $nR^2 = 10^2$ observable?
- low-energy finite-size limit: $\cot\delta_0 \to \text{const} \Rightarrow$ nontrivial $g(a_{2D})$, see [Zhang, Ho, J. Phys. B **51**, 115301 (2018)]
- second sound as a probe of BKT transition, see [Ozawa, Stringari, PRL **112**, 025302 (2014)] [AT, Cappellaro, Bighin, Salasnich, arXiv:2009.06491]

$\Rightarrow$ future work!

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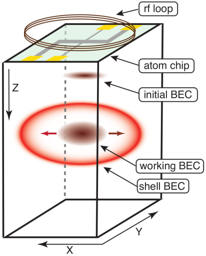
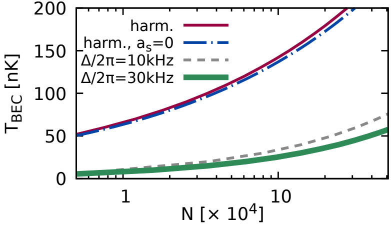
Properties and challenges of shell-shaped condensates

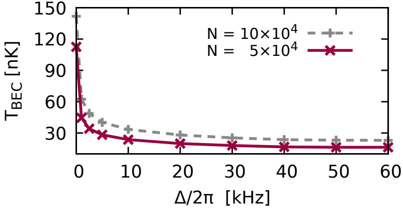
For the realistic trap parameters of NASA-JPL CAL experiment:

$$T_{BEC}^{bubble\ trap} \ll T_{BEC}^{harmonic\ trap}\ *$$



(*from Hartree-Fock theory
[Giorgini *et al.* J. Low T. Phys. (1997)])

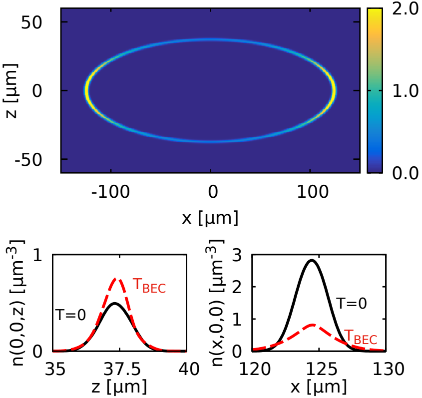




[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

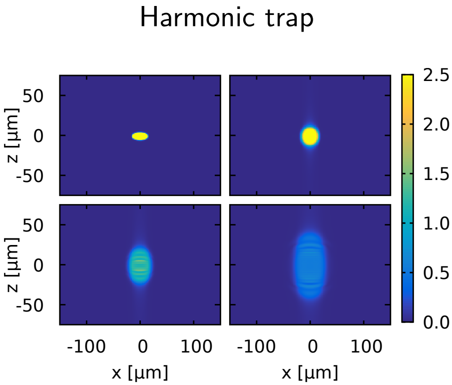
Number density at $T = 0$ and $T_{\rm BEC}$

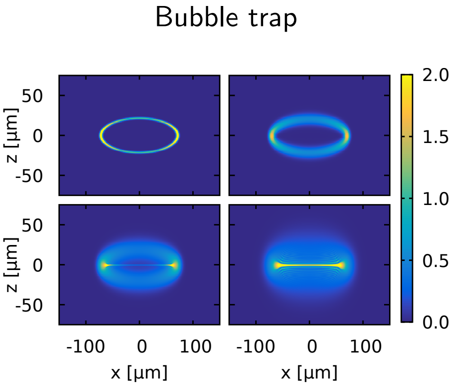
Density as a probe of the system temperature



[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

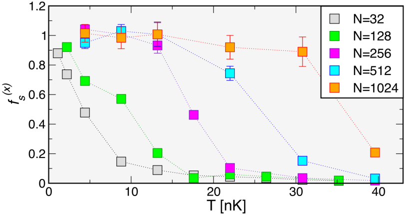
Free expansion



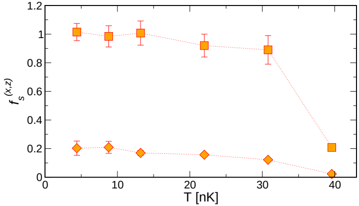


[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

Path Integral Monte Carlo - superfluid fraction



$f_s^{(x)}$ of strongly-interacting bosons on the ellipsoid



anisotropic $f_s^{(x,z)}$ from nonclassical moment of inertia

[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

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Summary and outlook

- I sketched how shell-shaped Bose-Einstein condensates are experimentally produced

- I analyzed T_{BEC}, T_{BKT}, and n_0/n for a spherical shell.
  $\rightarrow$ Further investigations on the BEC-BKT interplay

- Experiments can be challenging: to have a sufficient condensate fraction in $\sim 10^4$ atoms you need a final temperature $< 10\,\mathrm{nK}$.
  $\Rightarrow$ It is worth studying the finite-temperature properties

- Shell shaped Bose-Einstein condensates are a new configuration that the scientific community should start exploring

Thank you for your attention!

Main references:

A. Tononi and L. Salasnich, *Bose-Einstein Condensation on the Surface of a Sphere*, Phys. Rev. Lett. **123**, 160403 (2019).

A. Tononi, F. Cinti, and L. Salasnich, *Quantum Bubbles in Microgravity*, Phys. Rev. Lett. **125**, 010402 (2020).

in collaboration with

Luca Salasnich

Fabio Cinti

This presentation on www.andreatononi.com

Additional references

- N. Lundblad, R. A. Carollo, C. Lannert, et al. *Shell potentials for microgravity Bose-Einstein condensates*, npj Microgravity **5**, 30 (2019).
- D. C. Aveline, *et al.*, *Observation of Bose-Einstein condensates in an Earth-orbiting research lab*, Nature **582**, 193 (2020).
- K. Frye, *et al.*, *The Bose-Einstein Condensate and Cold Atom Laboratory*, arXiv:1912.04849
- K. Padavić, K. Sun, C. Lannert, and S. Vishveshwara, Phys. Rev. A **102**, 043305 (2020).
- K. Sun, K. Padavić, F. Yang, S. Vishveshwara, and C. Lannert, Phys. Rev. A **98**, 013609 (2018).