Equilibrium and nonequilibrium physics of bubble-trapped condensates

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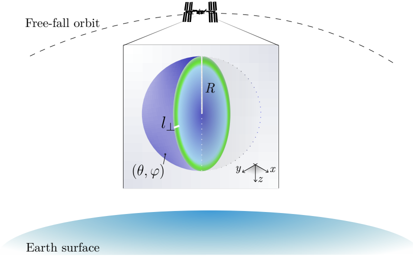

In collaboration with
F. Cinti, A. Pelster, L. Salasnich.

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A mini-talk/poster on

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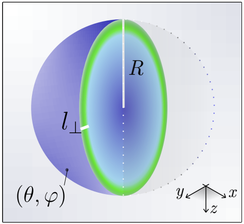
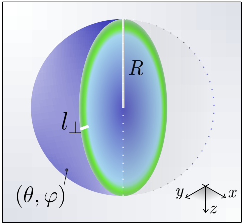
Bubble-trapped BECs



curved and hollow
2D Bose gas

A mini-talk/poster on

Bubble-trapped BECs



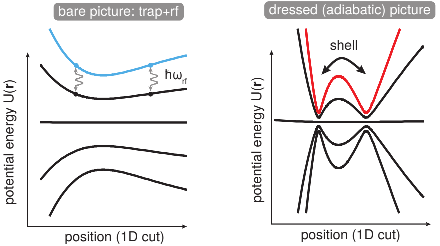
curved and hollow 2D Bose gas

Contents:

- Experiments — overview and numerical results
- Spherical shell — T_{BEC}, equation of state
- BKT transition
- Hydrodynamic excitations

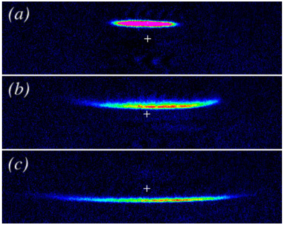
Experimentally realizable

Bubble-trap…



[Lundblad *et al.*, npj Microgravity **5**, 30 (2019)]

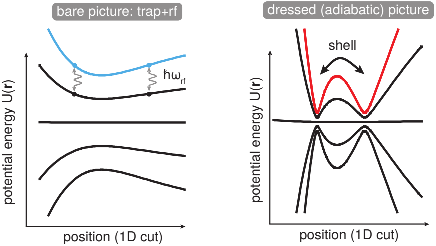
…on Earth



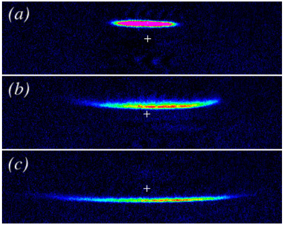
[Colombe *et al.*, EPL **67**, 593 (2004)]

Experimentally realizable...in microgravity

Bubble-trap... ...on Earth



[Lundblad *et al.*, npj Micrography **5**, 30 (2019)]

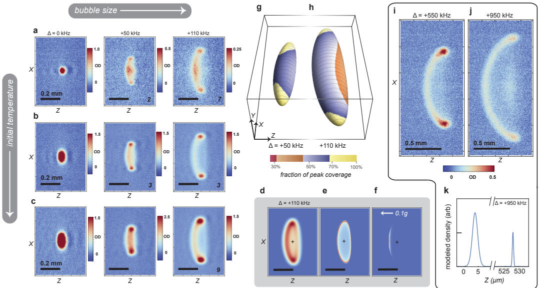


[Colombe *et al.*, EPL **67**, 593 (2004)]

⇒ Experiments on NASA
Cold Atom Lab

[Carollo *et al.*, arXiv:2108.05880]

[Aveline *et al.*, Nature **582**, 193 (2020)]



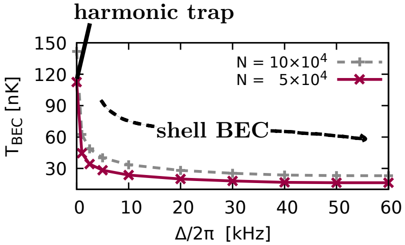
Numerical results — inflating the ellipsoidal bubble

Bubble-trap: $U(\vec{r}) = M_F \sqrt{\left[\sum_i m\,\omega_i{}^2 x_i^2/2 - \hbar\Delta\right]^2 + (\hbar\Omega)^2}$,

[Zobay, Garraway, Phys. Rev. Lett. **86**, 1195 (2001)]

For the realistic trap parameters of NASA-JPL CAL experiment:

T_{BEC} drops quickly with $\Delta \propto$ shell area



[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

Difficult to reach fully-condensate regime...

$\Rightarrow$ Finite-temperature properties and BKT physics are highly relevant

Numerical results — density at $T = 0$ and at $T_{\rm BEC}$

Number density as a probe of the system temperature

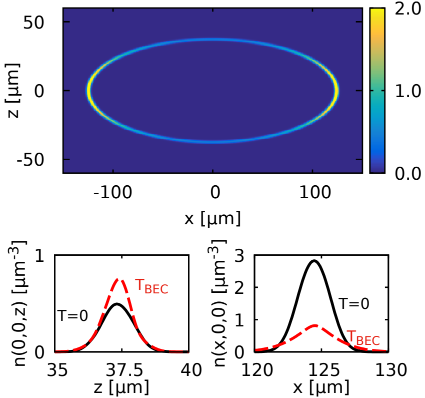
Gross-Pitaevskii equation at $T = 0$:

$$\left[-\frac{\hbar^2\nabla^2}{2m} + U(\vec{r}) + g|\psi|^2\right]\psi = \mu\psi$$

Hartree-Fock at $T_{\rm BEC}$:

$$n(\vec{r}) = \int \frac{d^3p}{e^{(E^{HF}(\vec{p},\vec{r}))/(k_B T_{\rm BEC})} - 1}$$

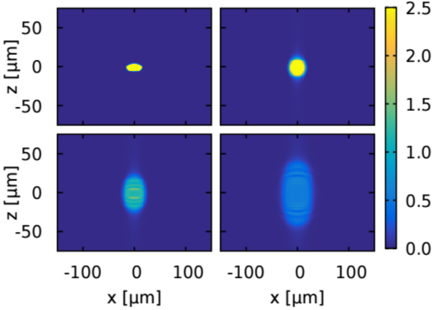
$$E^{HF}(\vec{p},\vec{r}) = p^2/(2m) + U(\vec{r}) - \mu + 2gn(\vec{r})$$



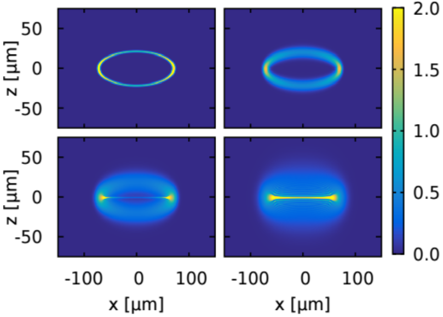
[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

Numerical results — free expansion

Harmonic trap



Bubble trap



[AT, Cinti, Salasnich, PRL **125**, 010402 (2020)]

Can we have analytical results?

Bose-Einstein condensation on the surface of a sphere

Can we have analytical results? $\rightarrow$ spherical symmetry

Noninteracting case, single particle on a sphere of radius R:

$\frac{\hat{L}^2}{2mR^2}\,\psi_{l,m_l}(\theta,\varphi) = \epsilon_l\,\psi_{l,m_l}(\theta,\varphi)$,

with $\epsilon_l = \frac{\hbar^2}{2mR^2}\,l(l+1)$ and $m_l = -l,\ldots,+l$.

Particle number at temperature T:

$$N = N_0 + \sum_{l=1}^{+\infty}\sum_{m_l=-l}^{+l}\frac{1}{e^{(\epsilon_l-\epsilon_0)/(k_B T)}-1}$$

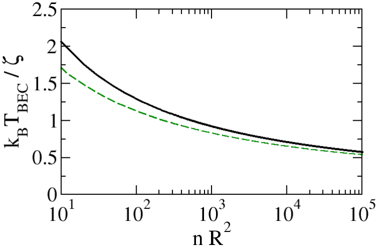
when $N_0 = 0 \implies T = T_{\text{BEC}}^{(0)}$

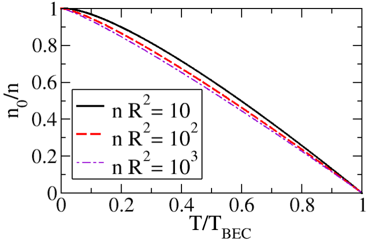
[AT, Salasnich, PRL **123**, 160403 (2019)]

BEC on a sphere: **noninteracting case**

$$k_B T^{(0)}_{\rm BEC} = \frac{\frac{2\pi\hbar^2}{m} n}{\frac{\beta^{(0)}_{\rm BEC}\hbar^2}{mR^2} - \ln\left(e^{\beta^{(0)}_{\rm BEC}\hbar^2/mR^2} - 1\right)}$$

$$\frac{n_0}{n} = 1 - \frac{1 - \frac{mR^2}{\hbar^2\beta}\ln\left(e^{\beta\hbar^2/mR^2} - 1\right)}{1 - \frac{mR^2}{\hbar^2\beta^{(0)}_{\rm BEC}}\ln\left(e^{\beta^{(0)}_{\rm BEC}\hbar^2/mR^2} - 1\right)}$$





[AT, Salasnich, PRL **123**, 160403 (2019)]

BEC on a sphere: **interacting case**

With Gaussian functional integration: interacting critical temperature

$$k_B T_{\rm BEC} = \frac{\frac{2\pi\hbar^2 n}{m} - \frac{gn}{2}}{\frac{\hbar^2 \beta_{BEC}}{2mR^2}\left(1 + \sqrt{1 + \frac{2gmnR^2}{\hbar^2}}\right) - \ln\left(e^{\frac{\hbar^2 \beta_{BEC}}{mR^2}\sqrt{1+\frac{2gmnR^2}{\hbar^2}}} - 1\right)}.$$

and the condensate fraction

$$\frac{n_0}{n} = 1 - \frac{mg}{4\pi\hbar^2} - \frac{1}{4\pi R^2 n}\left[1 + \sqrt{1 + \frac{2gmnR^2}{\hbar^2}}\right] + \frac{mk_B T}{2\pi\hbar^2 n}\ln\left(e^{\frac{\hbar^2}{mR^2 k_B T}\sqrt{1+(2gmnR^2/\hbar^2)}} - 1\right).$$

$R \to \infty$: $T_{\rm BEC} \to 0$, flat-case quantum depletion [Schick, PRA **3**, 1067 (1971)]

[AT, Salasnich, PRL **123**, 160403 (2019)]

Equation of state

Recent calculation of the finite-temperature grand potential

$$\frac{\Omega}{4\pi R^2} = -\frac{m\mu^2}{8\pi\hbar^2}\left[\ln\left(\frac{4\hbar^2}{m(E_1^B+\epsilon_1+\mu)a^2\, e^{2\gamma+1}}\right)+\frac{1}{2}\right]$$
$$+\frac{mE_1^B}{8\pi\hbar^2}(E_1^B-\epsilon_1-\mu)+\frac{1}{4\pi R^2}\frac{1}{\beta}\sum_{l=1}^{\infty}\sum_{m_l=-l}^{l}\ln\Big(1-e^{-\beta E_l^B}\Big),$$

[AT, Pelster, Salasnich, arXiv:2104.04585]

[AT, under review]

Equation of state

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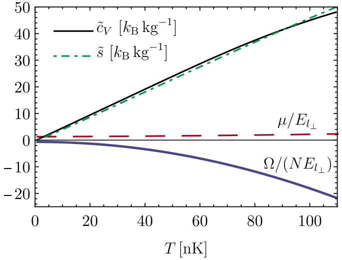
from which we calculate all the thermodynamic functions,

which will become useful to calculate hydrodynamic excitations...



[AT, Pelster, Salasnich, arXiv:2104.04585]

[AT, under review]

BKT transition of a 2D spherical superfluid

Kosterlitz-Nelson analysis to describe the superfluid transition yields

RG equations as in flat case

$$\frac{dK^{-1}(\theta)}{d\ell(\theta)} = -4\pi^3 y^2(\theta)$$

$$\frac{dy(\theta)}{d\ell(\theta)} = [2 - \pi K(\theta)]\, y(\theta)$$

RG scale?

$$\ell(\theta) = \ln[2R\sin(\theta/2)/\xi]$$

Distance between vortices:

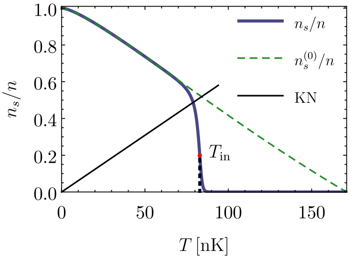
$2R\sin(\theta/2) \in [\xi, 2R]$...

...but in 3D space!!

[AT, Pelster, Salasnich, arXiv:2104.04585]

BKT transition of a 2D spherical superfluid

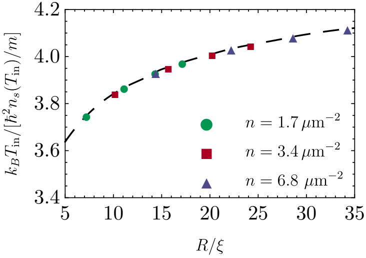
Finite system size
$\Rightarrow$ **smooth** vanishing of n_s

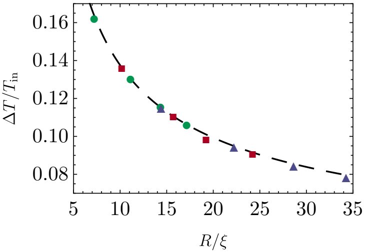


and finite-size universal BKT scaling:

$T_{\rm in} \sim n_s(T_{\rm in})$

$\Delta T/T_{\rm in} \propto \ln^{-2}(R/\xi)$



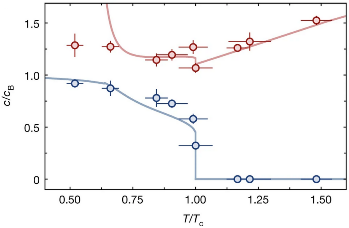


[AT, Pelster, Salasnich, arXiv:2104.04585]

Probing BKT physics with hydrodynamic modes

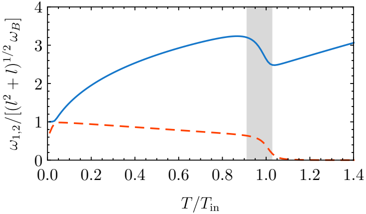
Response of a finite-temperature superfluid to a small perturbation:

Flat case:
ordinary **first and second sound**
(basis: plane waves $e^{i(kx-\omega t)}$)



[Christodoulou, et al. Nature **594**, 191 (2021)]

Shell BECs:
hydrodynamic modes ω_1, ω_2
(basis: spherical harmonics $\mathcal{Y}_l^{m_l} e^{i\omega t}$)



[AT, Pelster, Salasnich, arXiv:2104.04585]

The surface modes ω_1, ω_2 are the main quantitative probe of BKT physics

In conclusion

We investigate equilibrium and nonequilibrium properties of **2D bubble-trapped quantum gases**, either ellipsoidal, or spherical.

Interesting topics:

- BKT transition
- vortices

Understanding, probing, controlling the relation

quantum many-body physics $\leftrightarrow$ curvature

Thank you for your attention!

References

A. Tononi, F. Cinti, and L. Salasnich, *Quantum Bubbles in Microgravity*, Physical Review Letters **125**, 010402 (2020).

A. Tononi and L. Salasnich, *Bose-Einstein Condensation on the Surface of a Sphere*, Physical Review Letters **123**, 160403 (2019).

A. Tononi, A. Pelster, and L. Salasnich, arXiv:2104.04585