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Self-binding of one-dimensional fermionic mixtures

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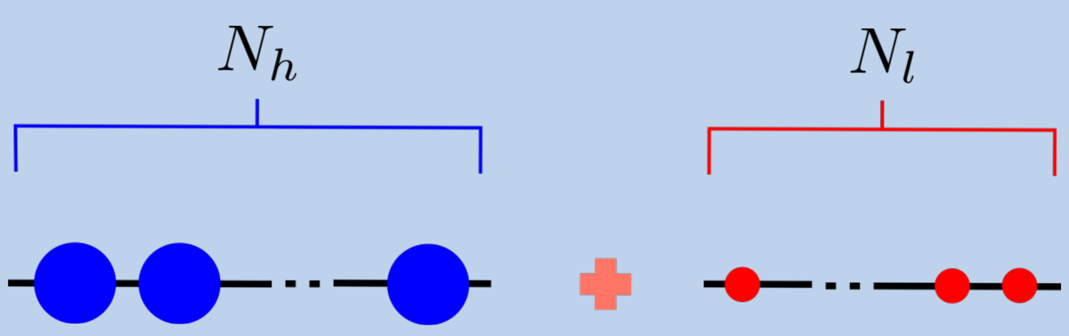
System

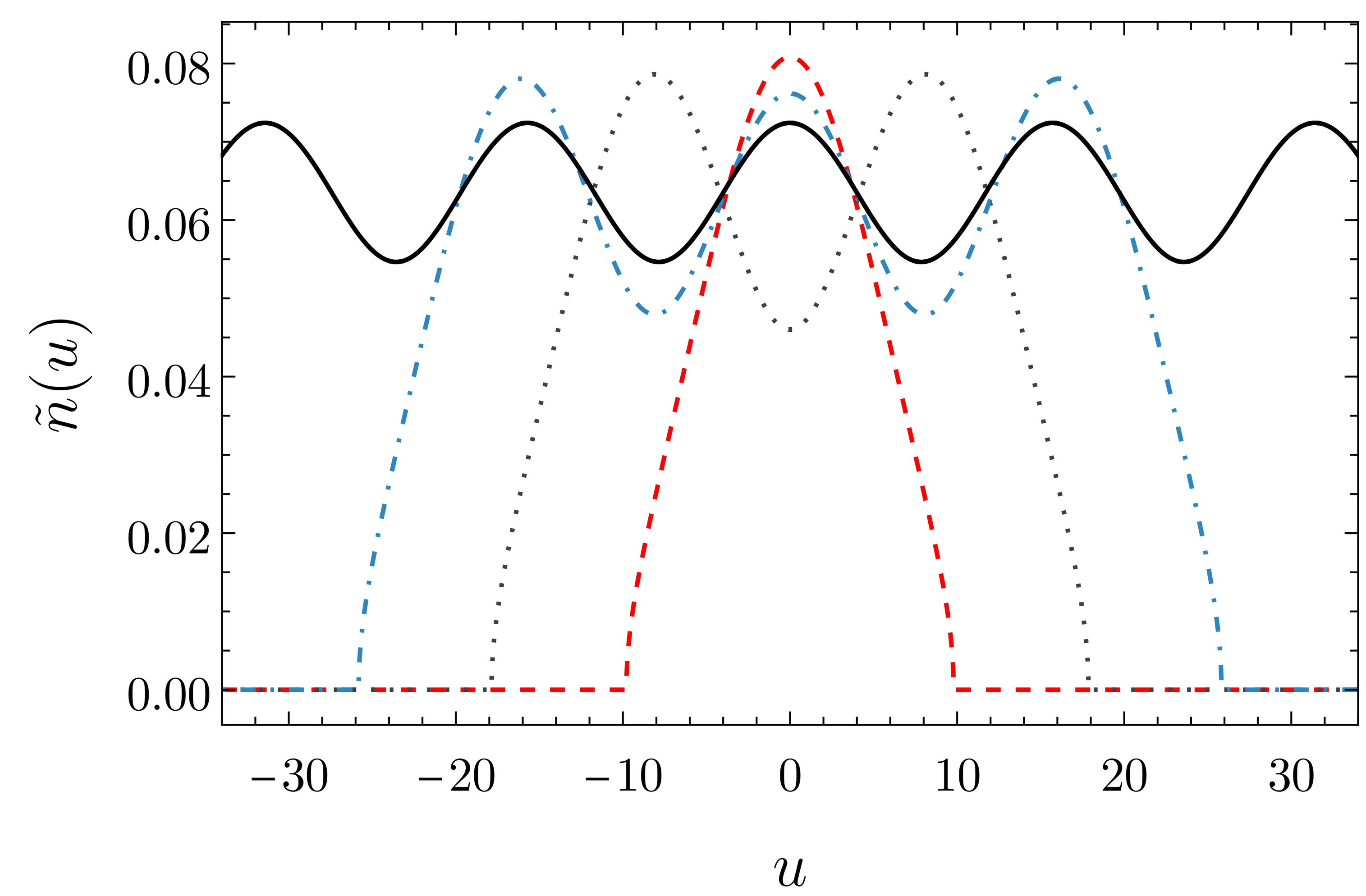
- N_h heavy fermions of mass M
- N_l light fermions of mass m
- Inter-species attraction (g<0)
- No intra-species interaction



Competition between the Pauli repulsion and the interspecies attraction

$$\hat{H} = \int \left(-\frac{\hat{\Psi}_x^\dagger \partial_x^2 \hat{\Psi}_x}{2M} - \frac{\hat{\phi}_x^\dagger \partial_x^2 \hat{\phi}_x}{2m} + g\hat{\Psi}_x^\dagger \hat{\phi}_x^\dagger \hat{\Psi}_x \hat{\phi}_x\right) dx$$

Question: what is the ground state of this system?

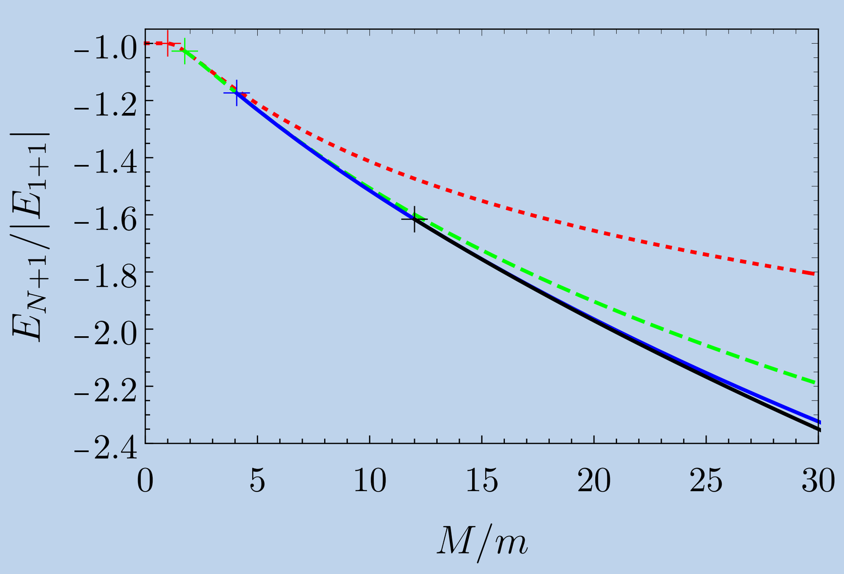


N+1 [1]

First off, let us start by taking **N_l = 1** and **N_h = N**

Finite N

We solve exactly the **Skorniakov-Ter Martirosian** equation and find the bound state energies for up to N = 5



Critical mass-ratio for binding:

$(M/m)_{2+1} = 1$
$(M/m)_{3+1} = 1.76$
$(M/m)_{4+1} = 4.2$
$(M/m)_{5+1} = 12.0 \pm 0.5$

Large N limit

We use a mean-field method, namely the **Thomas-Fermi** approximation for the kinetic energy of the heavy atom (in the limit where **N>>1**). The idea is to then write and minimize the grand potential

$$\Omega = \int \left[\frac{|\phi'(x)|^2}{2m} + gn(x)|\phi(x)|^2 + \frac{\pi^2 n^3(x)}{6M} - \epsilon|\phi(x)|^2 - \mu n(x)\right] dx$$

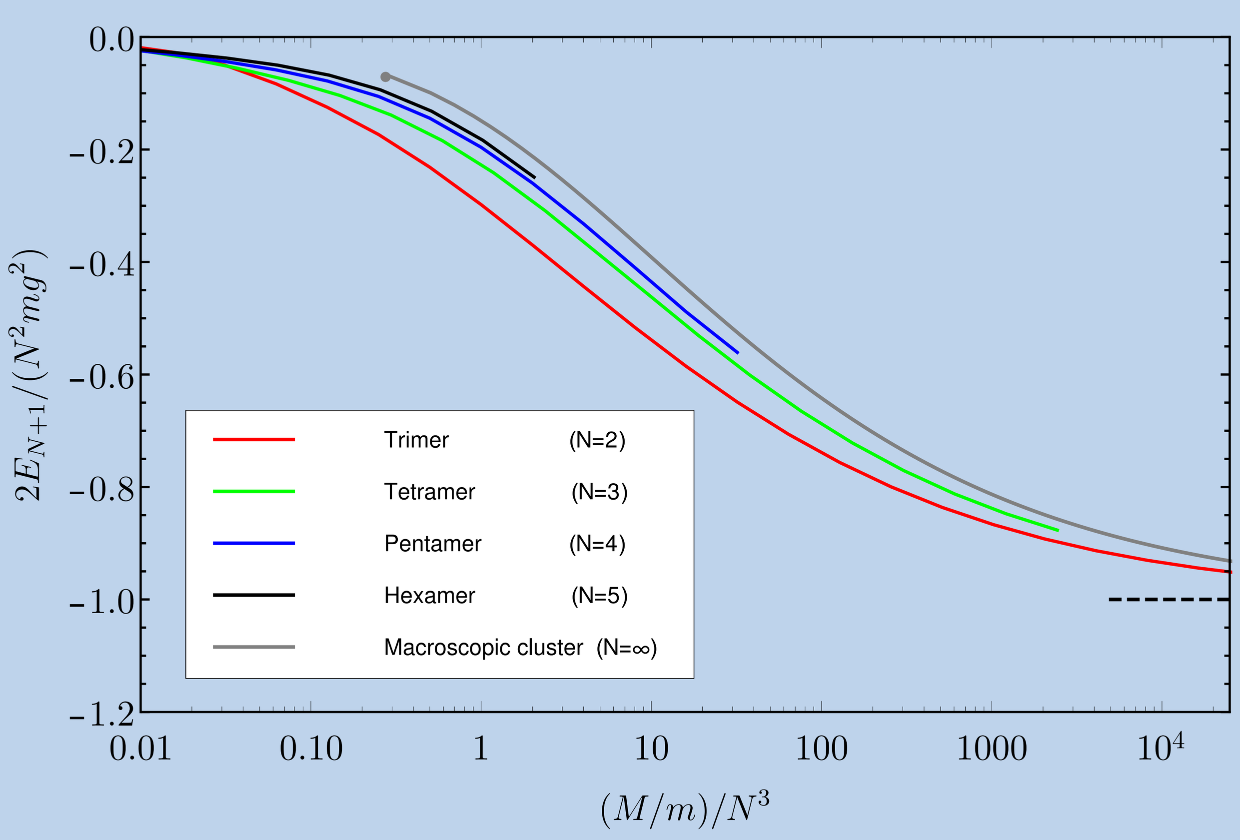
With normalization conditions

$$\int n(x)dx = N$$
$$\int \phi^2(x)dx = 1$$

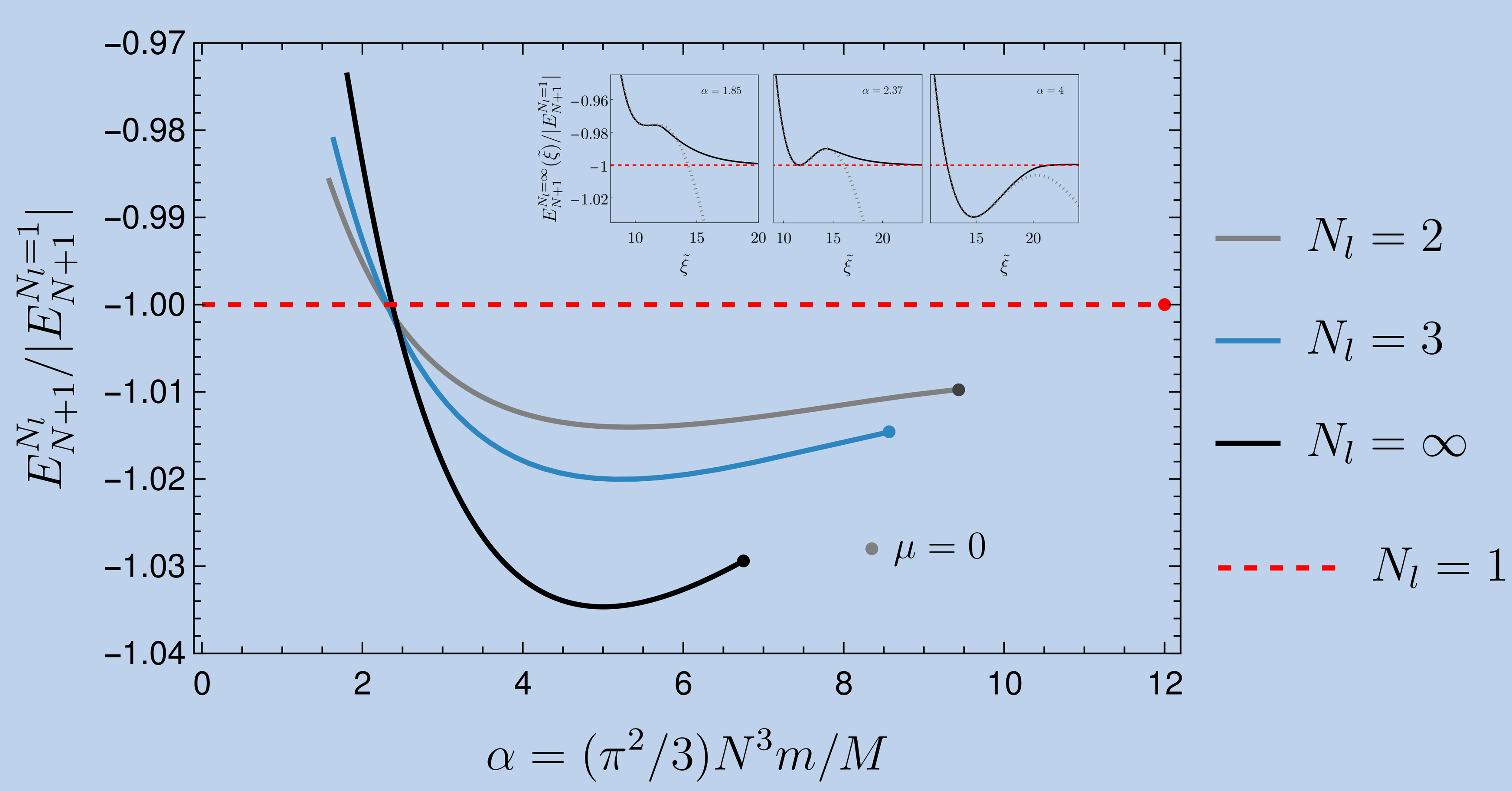
When the chemical potential is zero, it can be solved analytically and we find:

Critical mass-ratio for binding (μ=0):

$$(M/m)_{N+1} = \frac{\pi^2}{36} N^3$$



General case N_h + N_l [2]



We follow the same mean-field approach with the density functional Thomas-Fermi approximation. We also introduce the lengthscale $\lambda = 1/(2m|g|N)$ with $N=N_h/N_l$. We use then use this length scale to rescale the different quantities:

- $\phi_i(x) = \tilde{\phi}_i(u)$
- $n(x) = N\tilde{n}(u)$
- $u = x/\lambda$

Before

$$\Omega = \int dx \left[\sum_{i=1}^{N_l}\left(\frac{|\partial_x \phi_i|^2}{2m} + gn|\phi_i|^2\right) + \frac{\pi^2 n^3}{6M} - \sum_{i=1}^{N_l}\epsilon_i|\phi_i|^2 - \mu n\right]$$

With normalization conditions

$$\int n(x)dx = N_h$$
$$\int \phi_i(x)\phi_j(x)dx = \delta_{i,j}$$

After

$$\frac{\Omega}{2mg^2N^2} = \int \left[\sum_{i=1}^{N_l}\left(|\partial_u\tilde{\phi}_i|^2 - \tilde{n}|\tilde{\phi}_i|^2\right) + \alpha\tilde{n}^3 - \sum_{i=1}^{N_l}\tilde{\epsilon}_i|\tilde{\phi}_i|^2 - \tilde{\mu}\tilde{n}\right] du$$

With normalization conditions

$$\int \tilde{n}(u)du = 1$$
$$\int \tilde{\phi}_i^*(u)\tilde{\phi}_j(u)du = \delta_{ij}$$

Single controlling parameter:

$$\alpha = \frac{\pi^2 m N^3}{3M}$$

We also consider the infinite chain N_l>>1 in a similar manner and solve everything numerically using an iterative scheme and find the following binding threshold for α:

Predicted range of α for binding

$\alpha_{N_l=2} \in [1.6,\ 9.4]$
$\alpha_{N_l=3} \in [1.6,\ 8.6]$
$\alpha_{N_l=\infty} \in [1.8,\ 6.8]$

References :

[1] **Binding of heavy fermions by a single light atom in one dimension**, A. Tononi, J. Givois, D. S. Petrov, Phys. Rev. A 106, L011302 (2022)

[2] **Self-binding of one-dimensional fermionic mixtures with zero-range interspecies attraction**, J. Givois, A. Tononi, D. S. Petrov, arXiv:2207.04742