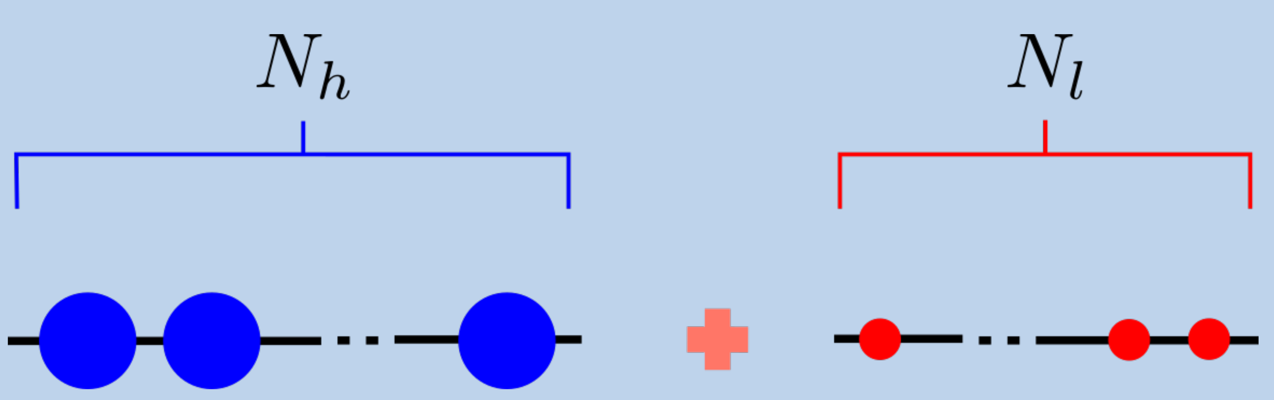
# Self-binding of one-dimensional fermionic mixtures


Jules Givois[1], Andrea Tononi[1], Dmitry S. Petrov[1]

[1] LPTMS, Université Paris-Saclay, 91405 Orsay, France


## System

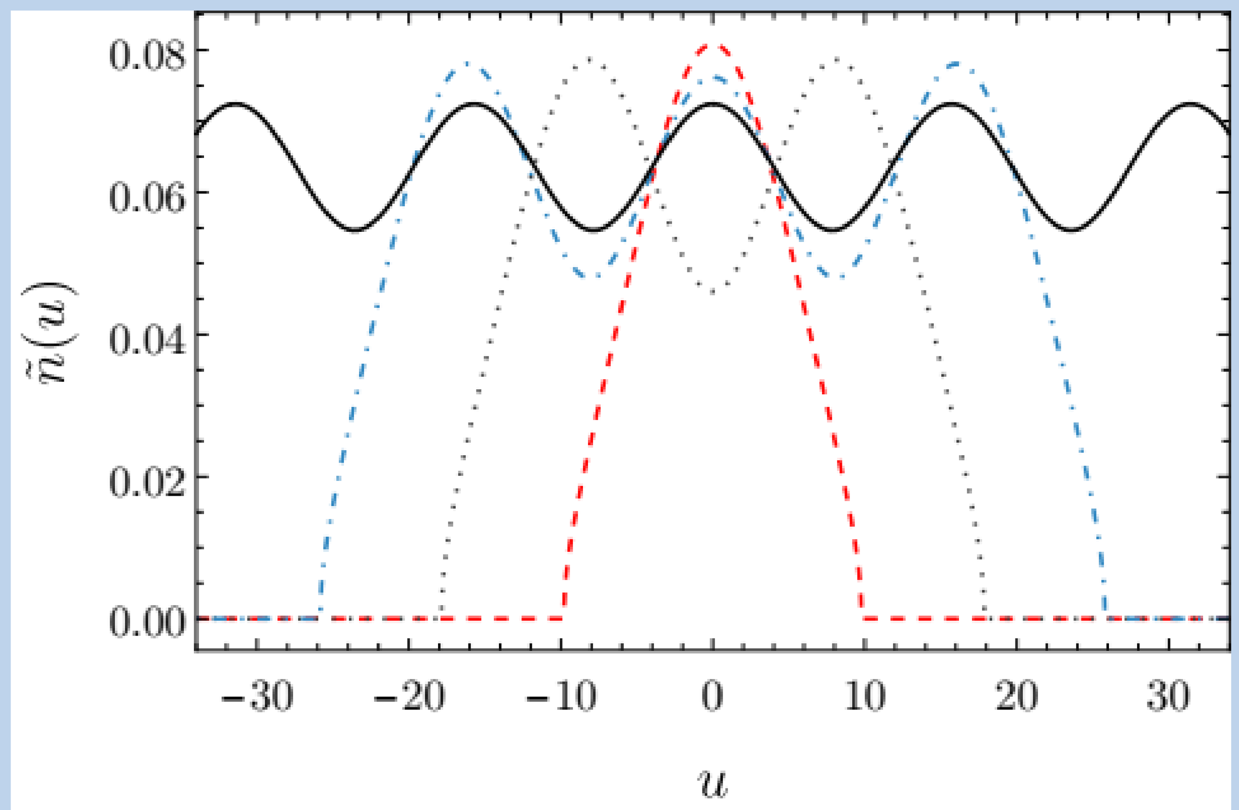
$$\hat{H} = \int \left( -\frac{\hat{\Psi}_x^\dagger \partial_x^2 \hat{\Psi}_x}{2M} - \frac{\hat{\phi}_x^\dagger \partial_x^2 \hat{\phi}_x}{2m} + g \hat{\Psi}_x^\dagger \hat{\phi}_x^\dagger \hat{\Psi}_x \hat{\phi}_x \right) dx$$




- $N_h$ heavy fermions, mass M
- $N_l$ light fermions, mass m
- Inter-species attraction (g<0)
- No intra-species interaction

Heavy atoms kinetic energy **VS** light-mediated heavy-heavy attraction.
What is the ground state?

Self-bound Fermi-Fermi mixture!

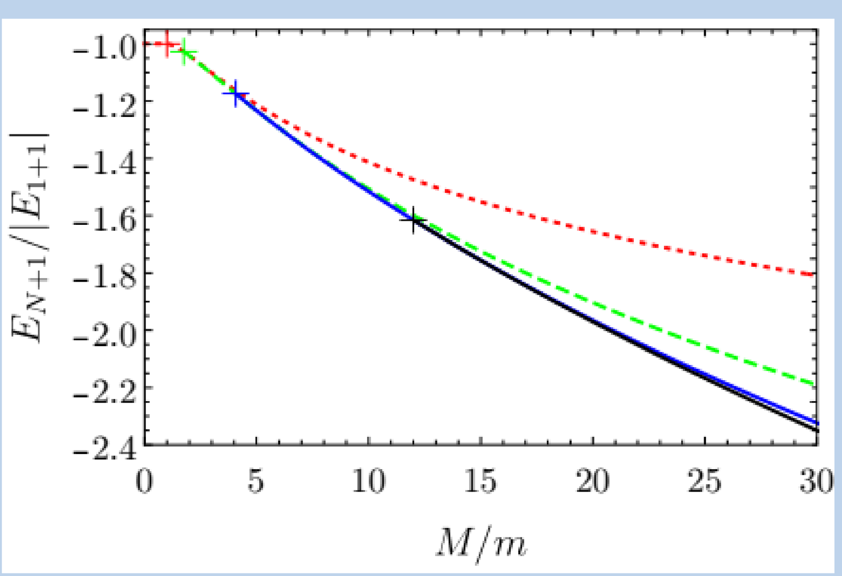


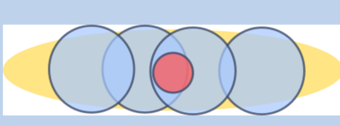

(in a region of the parameter space)
(n: spatial density of heavy atoms; see below)

## $N_h = N$ and $N_l = 1$ [1]

### Small N

Exact solution of the **Skorniakov-Ter Martirosian** equation.
Bound state energies up to N = 5:

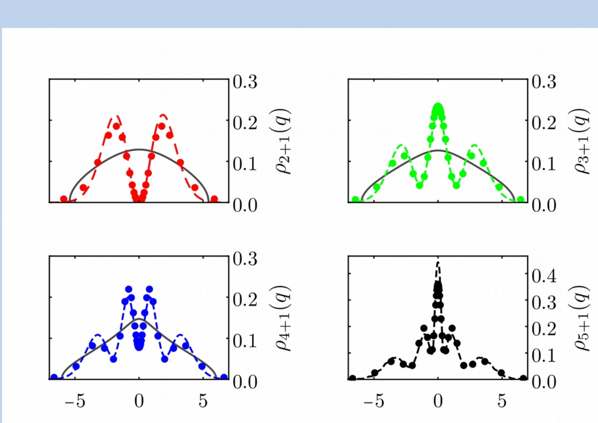


**Critical mass-ratio for binding**:

$(M/m)_{2+1} = 1$
$(M/m)_{3+1} = 1.76$
$(M/m)_{4+1} = 4.2$
$(M/m)_{5+1} = 12.0 \pm 0.5$

Few-body clusters of an Yb-Li mixture, momentum density




### Large N

**Thomas-Fermi approximation** for the kinetic energy of heavy atoms (mean field, **N>>1**).

We minimize the grand potential

$$\Omega = \int \left[ \frac{|\phi'(x)|^2}{2m} + g n(x) |\phi(x)|^2 + \frac{\pi^2 n^3(x)}{6M} - \epsilon |\phi(x)|^2 - \mu n(x) \right] dx$$

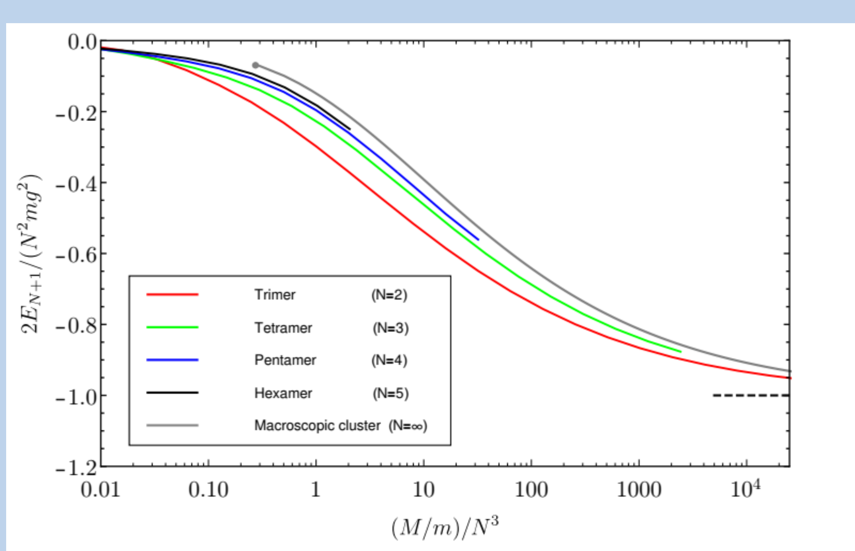
with normalization conditions

$$\int n(x) dx = N \qquad \int |\phi(x)|^2 dx = 1$$

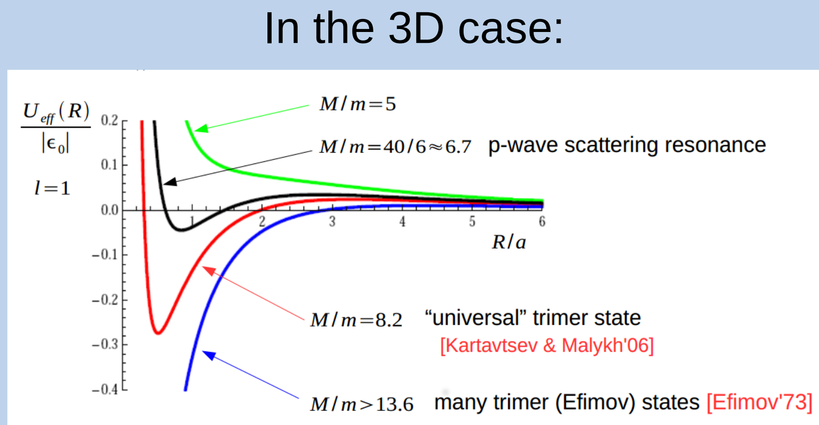
**Critical mass-ratio for binding (μ=0)**:

$$(M/m)_{N+1} = \frac{\pi^2}{36} N^3$$

TF approximation works for large N




Why N increases with the mass ratio?
In the 3D case:




## $N_h + N_l$ [2]

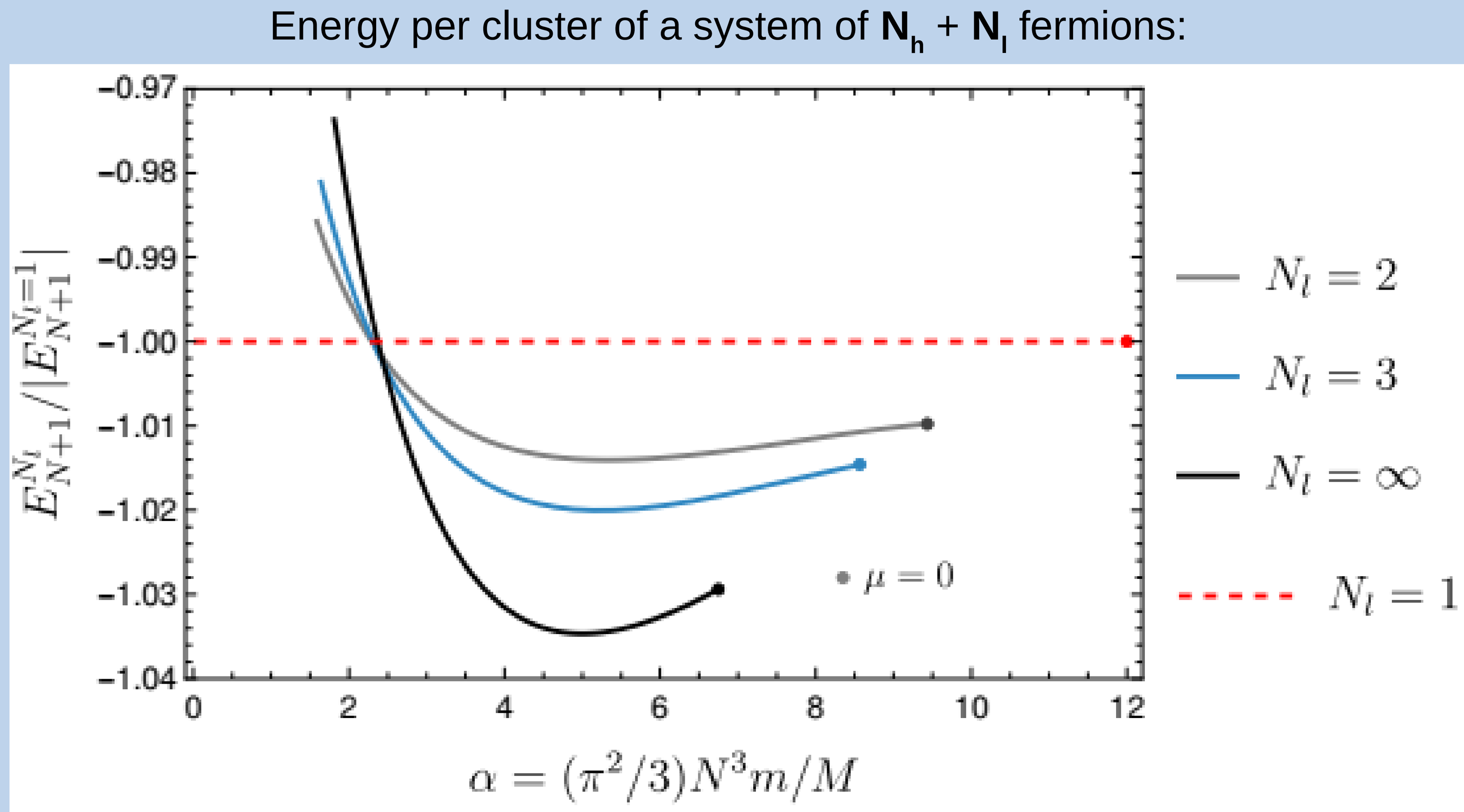
$$\frac{\Omega}{2mg^2N^2} = \int \left[ \sum_{i=1}^{N_l} \left( |\partial_u \tilde{\phi}_i|^2 - \tilde{n} |\tilde{\phi}_i|^2 \right) + \alpha \tilde{n}^3 - \sum_{i=1}^{N_l} \tilde{\epsilon}_i |\tilde{\phi}_i|^2 - \tilde{\mu} \tilde{n} \right] du$$

(rescaled with the length $\lambda = 1/(2m|g|N)$)

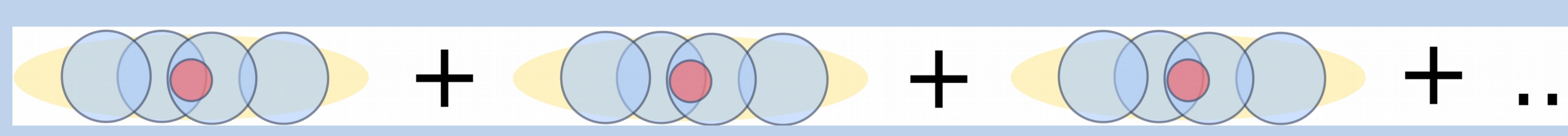
Only two parameters:

$$N_l, \quad \alpha = \frac{\pi^2 m N^3}{3M}$$

Energy per cluster of a system of $N_h$ + $N_l$ fermions:



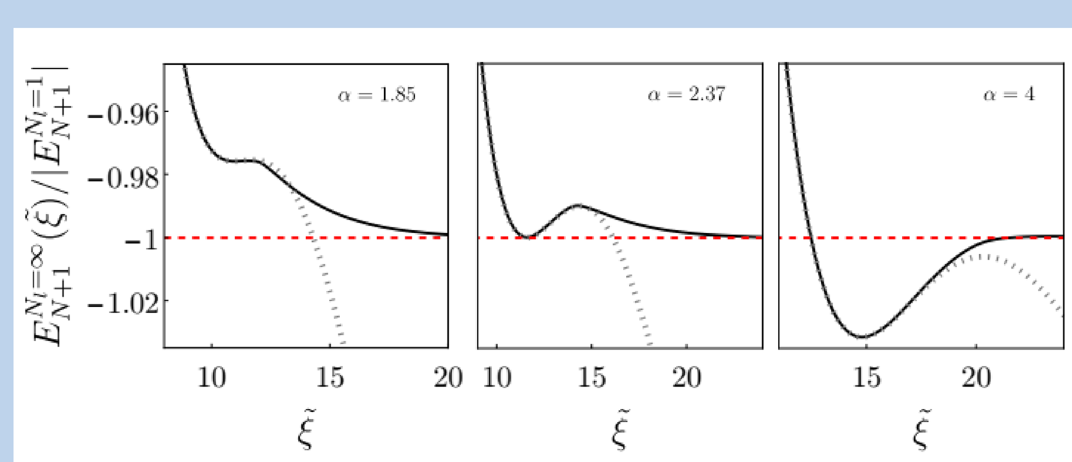

(each cluster contains **N+1** atoms, with **N=N_h/N_l**)



**Range of α for the self-binding**

$\alpha_{N_l=2} \in [1.6,\ 9.4] \quad \alpha_{N_l=3} \in [1.6,\ 8.6] \quad .... \quad \alpha_{N_l=\infty} \in [1.8,\ 6.8]$

### Infinite chain ($N_l$>>1)




Set 1 light atom per spatial period ξ.

Numerical minimization of the grand potential gives the energy as a function of ξ.


**References:**

[1] A. Tononi, J. Givois and D. S. Petrov, **Binding of heavy fermions by a single light atom in one dimension**, Phys. Rev. A **106**, L011302 (2022).

[2] J. Givois, A. Tononi and D. S. Petrov, **Self-binding of one-dimensional fermionic mixtures with zero-range interspecies attraction**, arXiv:2207.04742