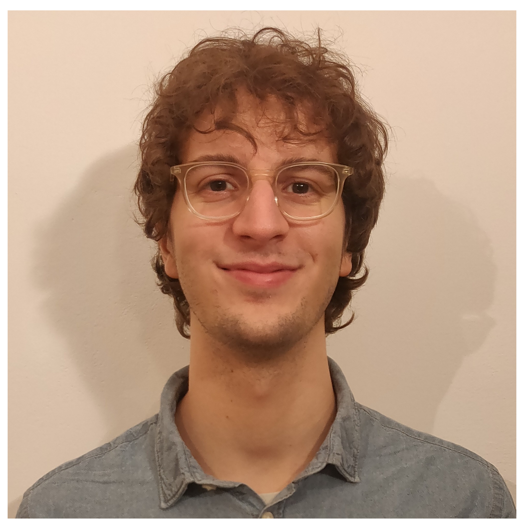

Condensation in 2D: from non-universal effects to bubble traps

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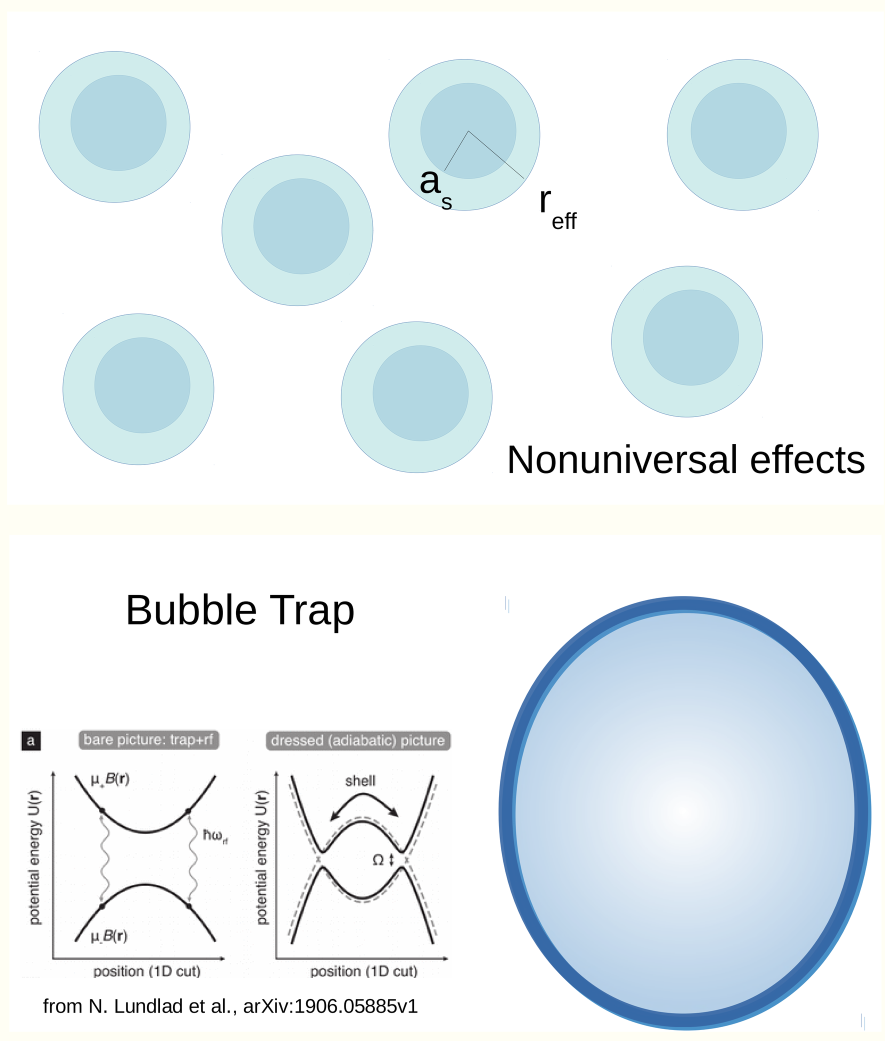
Graphical abstract



Bibliography

[1] A. Tononi, A. Cappellaro, L. Salasnich, New J. of Phys., **20**, 125007 (2018).

[2] A. Tononi, Condens. Matter **4**, 20 (2019).

[3] A. Tononi, L. Salasnich, Physical Review Letters, 123, 160403 (2019).

Functional integration

We consider a uniform D-dimensional Bose gas of identical cold atoms with mass m, described by the complex field $\psi(\vec{r},\tau)$.

The partition function

$$\mathcal{Z} = \int D(\bar{\psi},\psi)\, e^{-\frac{S[\bar{\psi},\psi]}{\hbar}},$$

$$S[\bar{\psi},\psi] = \int_0^{\beta\hbar} d\tau \int_V d^D r\, \mathcal{L},$$

$$\mathcal{L} = \bar{\psi}_{\vec{r},\tau}\left(\hbar\partial_\tau - \frac{\hbar^2\nabla^2}{2m} - \mu\right)\psi_{\vec{r},\tau} + \frac{1}{2}\int d^D r'\, |\psi_{\vec{r},\tau}|^2\, V(\vec{r}-\vec{r}\,')\, |\psi_{\vec{r}\,',\tau}|^2$$

Performing functional integration at a Gaussian level ($\psi_{\vec{r},\tau} = \psi_0 + \eta_{\vec{r},\tau}$), one obtains $\Omega = -\beta^{-1}\ln\mathcal{Z}$ as

$$\Omega(\mu,\psi_0^2,T) = \Omega_0 + \Omega_g^{(0)} + \Omega_g^{(T)}$$

with the excitation spectrum

$$E_{\vec{k}} = \sqrt{\left(\varepsilon_k - \mu + g_0\psi_0^2 + \psi_0^2\,\tilde{V}(\vec{k})\right)^2 - \left(\psi_0^2\,\tilde{V}(\vec{k})\right)^2}$$

From nonuniversal effects to bubble traps

Nonuniversal effects Weakly-interacting bosons are usually described with the zero-range interaction $\tilde{V}(k) = g_0$. With scattering theory, g_0 is linked to the s-wave scattering length a_s (universal parameter). We study $\tilde{V}(k) = g_0 + g_2 k^2$, including the first nonzero correction in the momentum. Here g_0 and g_2 are linked to a_s and:

- r_{eff} in 3D (effective range)
- R_{ch} in 2D (characteristic range)

In specific regimes (in 3D: $a_s/r_{eff} \lesssim 1$, in 2D: $a_s/R_{ch} \lesssim 1$) the finite-range corrections to the thermodynamics are relevant.

Bubble traps Bubble traps are produced with the external potential

$$U(\vec{r}) = \sqrt{(u(\vec{r}) + \hbar\Delta)^2 + \hbar\omega^2},$$

with $u(\vec{r})$ an harmonic potential. In the thin-shell limit, we simply put $V = S^2$.

Nonuniversal equation of state in 2D

With the superfluid parametrization $\psi = \sqrt{\rho_0 + \delta\rho}\, e^{i\theta}$, we derive in Ref. [2] the zero-temperature equation of state for a 2D homogeneous Bose gas

$$P(\mu, T=0) = \frac{m\mu^2}{8\pi\hbar^2\lambda^{3/2}}\left[\ln\left(\frac{\epsilon_0}{\mu}\lambda\right) - \frac{1}{2}\right], \qquad \lambda = 1 + \frac{4m\,\mu}{\hbar^2}\frac{g_2}{g_0}$$

For $na_s^2 = 10^{-5}$ and $nR_{ch}^2 = 6\times10^{-2}$ we get a **20%** correction.

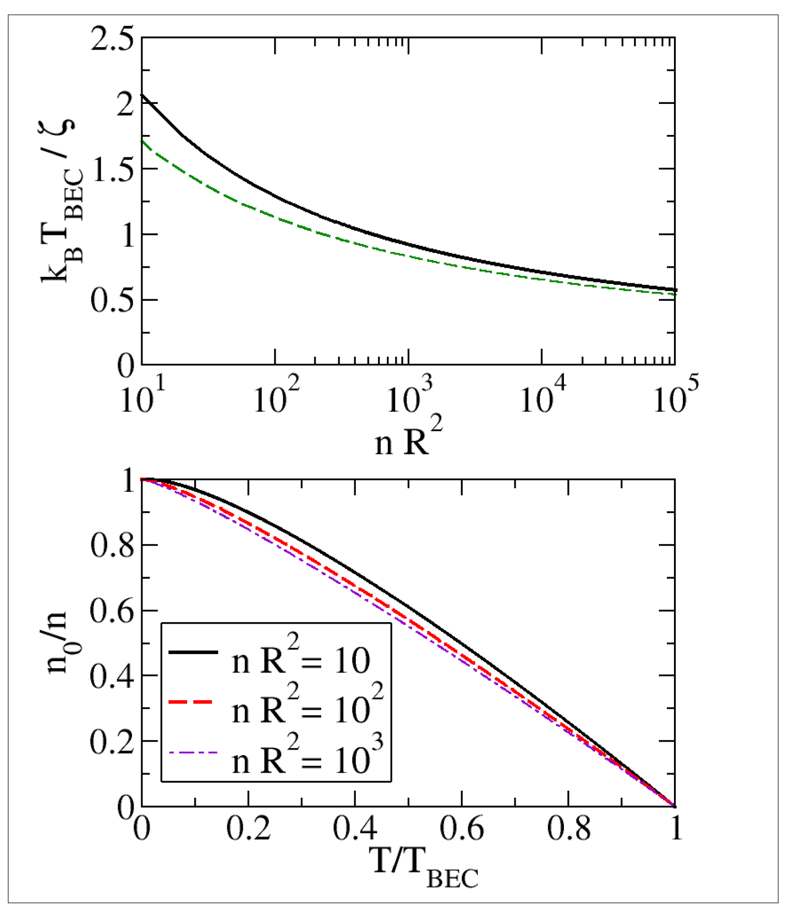
BEC on a sphere

In Ref. [3], we apply VPT to calculate the critical temperature T_{BEC} for interacting bosons confined on a 2D spherical shell

$$k_B T_{BEC} = \left(\frac{2\pi\hbar^2 n}{m} - \frac{gn}{2}\right) \Bigg/ \left[\frac{\hbar^2\beta_{BEC}}{2mR^2}\left(1 + \sqrt{1 + \frac{2gmnR^2}{\hbar^2}}\right) - \ln\left(e^{\frac{\hbar^2\beta_{BEC}}{mR^2}\sqrt{1+\frac{2gmnR^2}{\hbar^2}}} - 1\right)\right],$$

and the condensate fraction

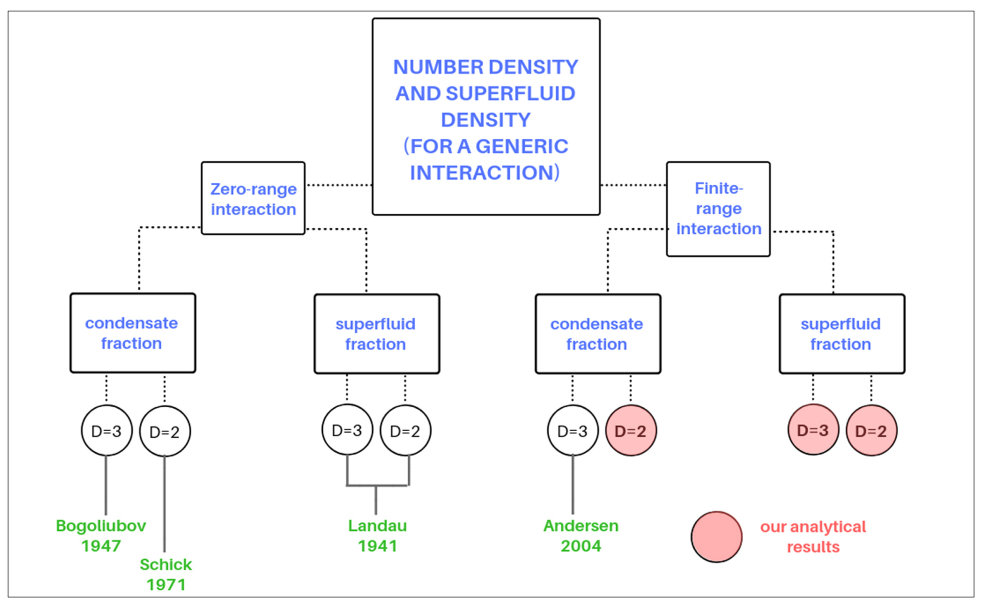
$$\frac{n_0}{n} = 1 - \frac{mg}{4\pi\hbar^2} - \frac{1}{4\pi R^2 n}\left[1 + \sqrt{1 + \frac{2gmnR^2}{\hbar^2}}\right] + \frac{mk_BT}{2\pi\hbar^2 n}\times \ln\left(e^{\frac{\hbar^2}{mR^2k_BT}\sqrt{1+\frac{2gmnR^2}{\hbar^2}}} - 1\right).$$



In the limit $R \to \infty$ one has $T_{BEC} \to 0$, and n_0/n gives Schick result (PRA,3,1067).
We calculate n_s in analogy to the Landau formula, and obtain T_{BKT} applying the Kosterlitz-Nelson criterion, unchanged with respect to the flat plane (PRD,43,1314).

The static and dynamic properties of bosons in bubble-traps will be studied in the ISS (microgravity conditions).

Condensation and superfluidity in D dimensions



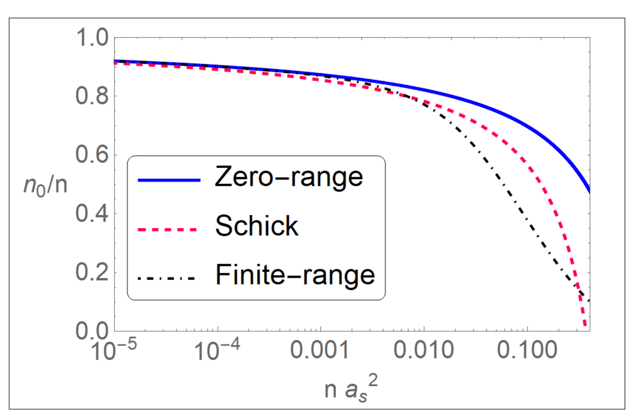
For the finite-range effective interaction $\tilde{V}(k) = g_0 + g_2 k^2$ in Ref. [1] we calculate

- the number density n as a function of condensate density n_0 and T as

$$n(n_0, T) = -\frac{1}{L^D}\frac{\partial\Omega(\mu_e,\psi_0,T)}{\partial\mu_e} = n_0 + f_g^{(0)} + f_g^{(T)}$$

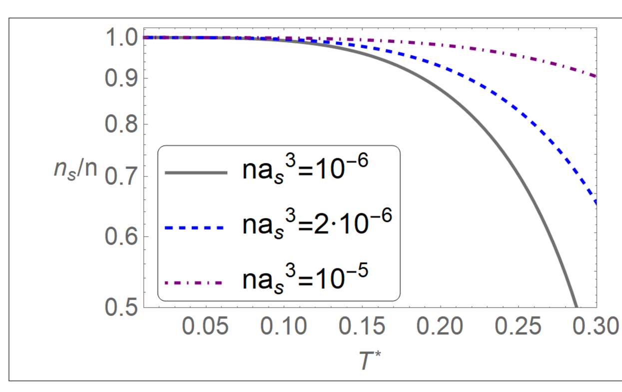
- the superfluid density $n_s = n(n_0,T) - n_n(n_0,T)$, with n_n the normal density given by a self-consistent derivation of Landau formula

Finite-range corrections to n_0/n



2D condensate fraction n_0/n at $T=0$ and $R_{ch} = 2a_s$, in terms of the gas parameter na_s^2.

Finite-range corrections to n_s/n



3D superfluid fraction n_s/n for $r_{eff} = a_s$, as a function of the temperature $T^* = k_BT/E_r$, where $E_r = \hbar^2 n^{2/3}/m$.

Contact informations

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